Theory

of

Correlation Method

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* Turbulent transport mechanism in the atmospheric surface layer
* Gradient method (profile method)
* Eddy correlation method (covariance method)
* indirect method destruction rate of conc. fluctuation by molecular diffusion

Mass Flux in vertical direction: Nz

$$Nz = -WC \quad - \langle wc \rangle \quad - D\,\partial C/\partial z$$

convection by mean velocity — convection by fluctuating velocity — molecular diffusion

$$\text{where } (\tilde{W}, \tilde{C}) = (W, C) + (w, c)$$

instantaneous — mean — fluctuation

W, w : vertical velocity

D : molecular diffusion coefficient

C, c : concentration

z : distance in upward direction

$\langle\,\rangle$: time average

$D\ \partial C/\partial z$: negligible

→ N_z is independent of diffusion coefficient

At the tropopause

$$N_z \sim -WC$$

Near the surface $W=0$

$$N_z \sim -\langle wc \rangle$$

Momentum flux τ_z :

$$\tau_z/\rho = -\langle uw \rangle \equiv (u*)^2$$

Heat Flux q_z :

$$q_z/\rho c_p = -\langle w\theta \rangle \equiv u*\ T*$$

Mass flux N_z:

$$N_z = -\langle wc \rangle \equiv u*\ C*$$

Methods for flux measurement

1. Gradient method (profile method)

Boussinesq concept of

eddy diffusivity K_d

$$N_z = -\langle wc \rangle = K_d \partial C / \partial z = u_* C_*$$

$$-\langle w\theta \rangle = K_d \partial \Theta / \partial z = u_* T_*$$

$$-\langle uw \rangle = K_m \partial U / \partial z = (u_*)^2$$

Near the surface level

1. u_*, T_*, C_* ; constant
2. $K_d = 1.2 \kappa u_* z / \phi_h$

   $K_m = \kappa u_* z / \phi_m$

   κ : Kármán constant (=0.40)

Shear functions, ϕ_m, ϕ_h

$$\phi_m \equiv (\kappa z / u_*) \, dU/dz,$$

$$\phi_h \equiv (\kappa z / T_*) \, d\Theta/dz$$

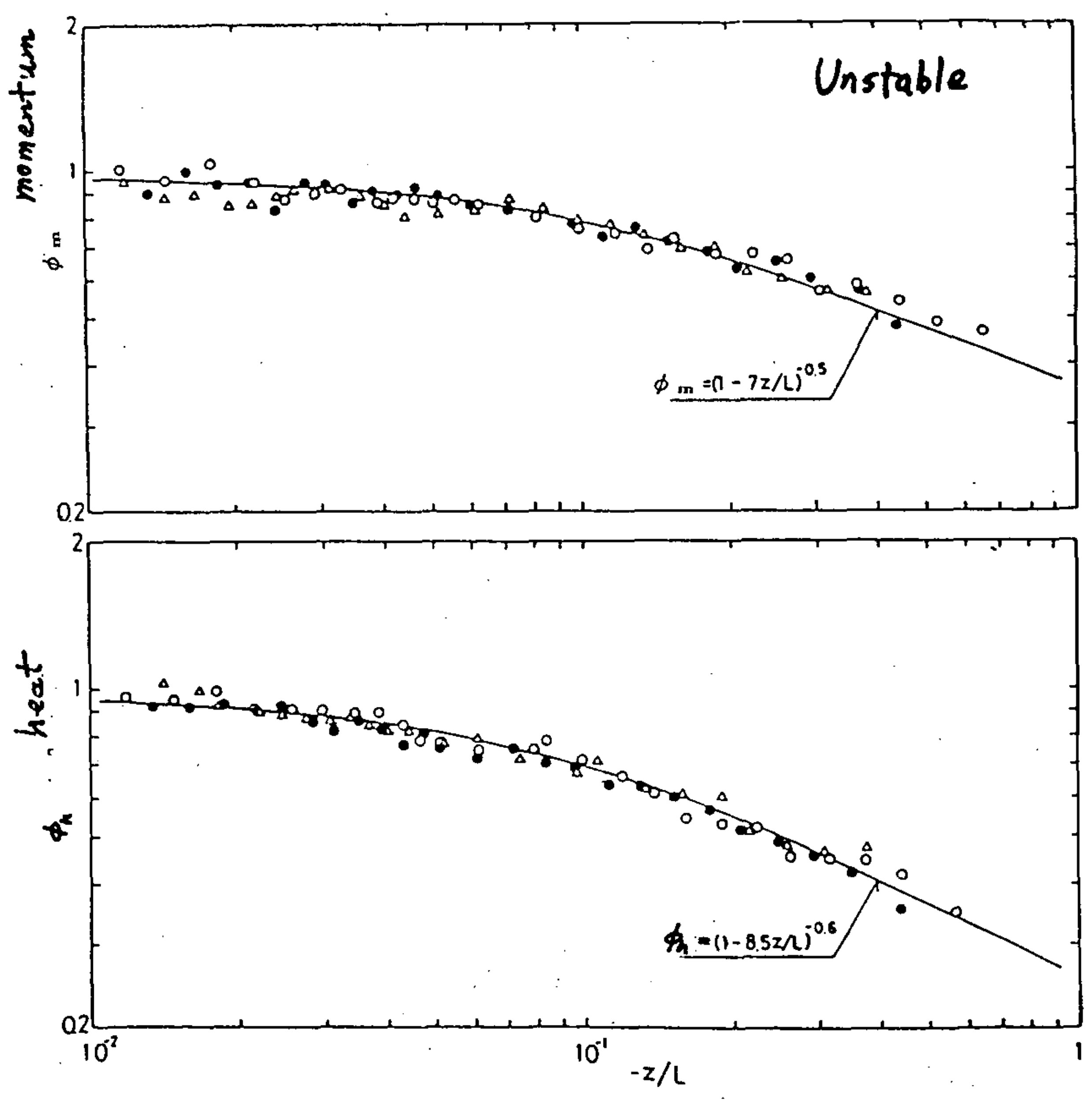
are universal function of z/L. stratification level, atmospheric stability

L : Monin−Obukhov length

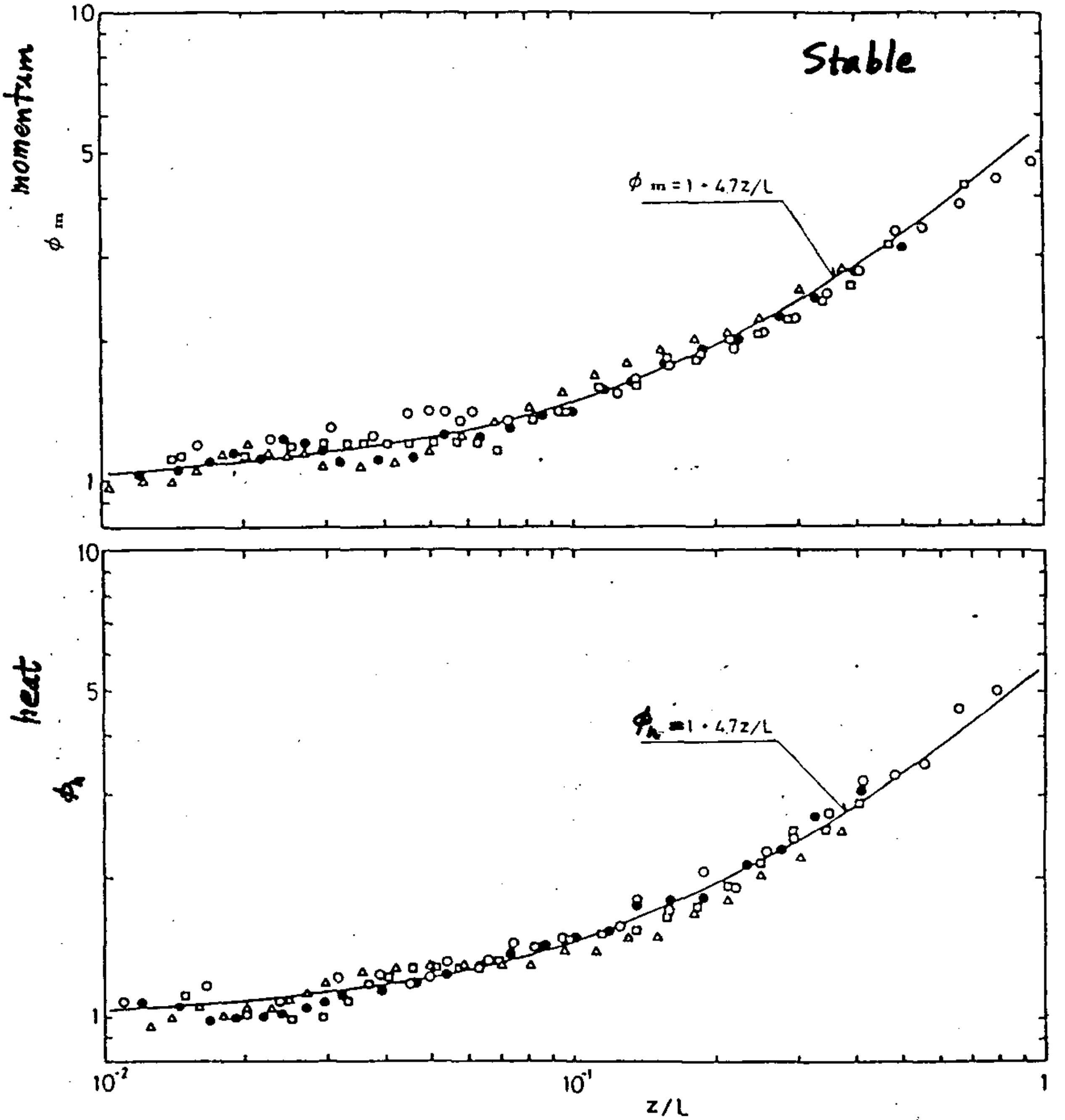
$$\equiv -(u_*)^3 \Theta_{ref} / (\kappa g \underbrace{\langle w\theta \rangle}_{u_* T_*})$$

Neutral stratification (adiabatic)

$z/L = 0 \quad (L \to \pm\infty)$



△, $Re = 8\,800$, $\overline{Ri} = -0{\cdot}0126$; ●, $Re = 9\,840$, $\overline{Ri} = -0{\cdot}0146$;
○, $Re = 7\,700$, $\overline{Ri} = -0{\cdot}0241$.



△, $Re = 6\,600$, $\overline{Ri} = 0{\cdot}0122$; ●, $Re = 6\,100$, $\overline{Ri} = 0{\cdot}0140$;
□, $Re = 6\,900$, $\overline{Ri} = 0.0270$; ○, $Re = 5\,500$, $\overline{Ri} = 0{\cdot}0514$.

$$\phi_m \equiv \frac{\kappa z}{u_*}\frac{dU}{dz}, \quad \phi_h = \frac{\kappa' z}{T^*}\frac{d\Theta}{dz} = \frac{\kappa' z}{C^*}\frac{dC}{dz} \quad (\kappa' = 1.2\kappa)$$

Integration

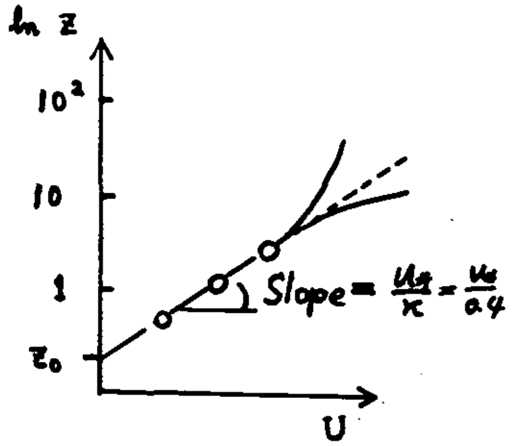
$$\frac{U}{u_*} = \frac{1}{\kappa z}\int_{z_0}^{z}\frac{1}{\kappa z}\phi_m\, dz, \quad \frac{C - C_0}{C^*} = \int_{z_{0d}}^{z}\frac{1}{\kappa' z}\phi_h\, dz$$

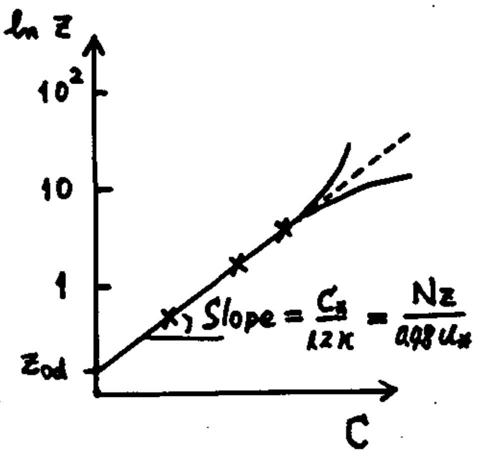
When $\phi_m = \quad \phi_h = 1 + \alpha(z/L)$

$$U/u_* = 1/\kappa\ [\ln z/z_0 + \alpha(z - z_0)/L]$$

$$(C - C_0)/C_* = 1/(1.2\kappa)\ [\ln z/z_{0d} + \alpha(z - z_{0d})/L]$$

$$(\Theta - \Theta_0)/T_* = 1/(1.2\kappa)\ [\ln z/z_{0h} + \alpha(z - z_{0h})/L]$$





$z \to 0$ or $L \to \pm\infty$

$$U/u_* = \frac{1}{\kappa}\ln\frac{z}{z_0}, \quad \frac{C - C_0}{C_*} = \frac{1}{1.2\kappa}\ln\frac{z}{z_{0d}}$$

Eddy correlation method (covariance method)

$$N_z = -\langle wc \rangle = -\frac{1}{T}\int_0^T wc\, dt = -\frac{1}{N}\sum_{i=1}^{N}(\tilde{W}_i - W)(\tilde{C}_i - C)$$

T: averaging time

$\tilde{W}_i$, $\tilde{C}_i$: instantaneous values (digitized)

$(W, C) = (1/N)\sum^{N}(\tilde{W}_i, \tilde{C}_i)$ mean

Instruments : high time resolution (frequency response)
high spatial resolution
rain, snow etc.

Vertical velocity component
- Sonic anemometer thermometer ($f_m = 10 - 20\,Hz$)
- Vane anemometer ($f_m \sim 10\,Hz$)
- Hot-wire anemometer ($f_m = 10^3 \sim 10^4\,Hz$)

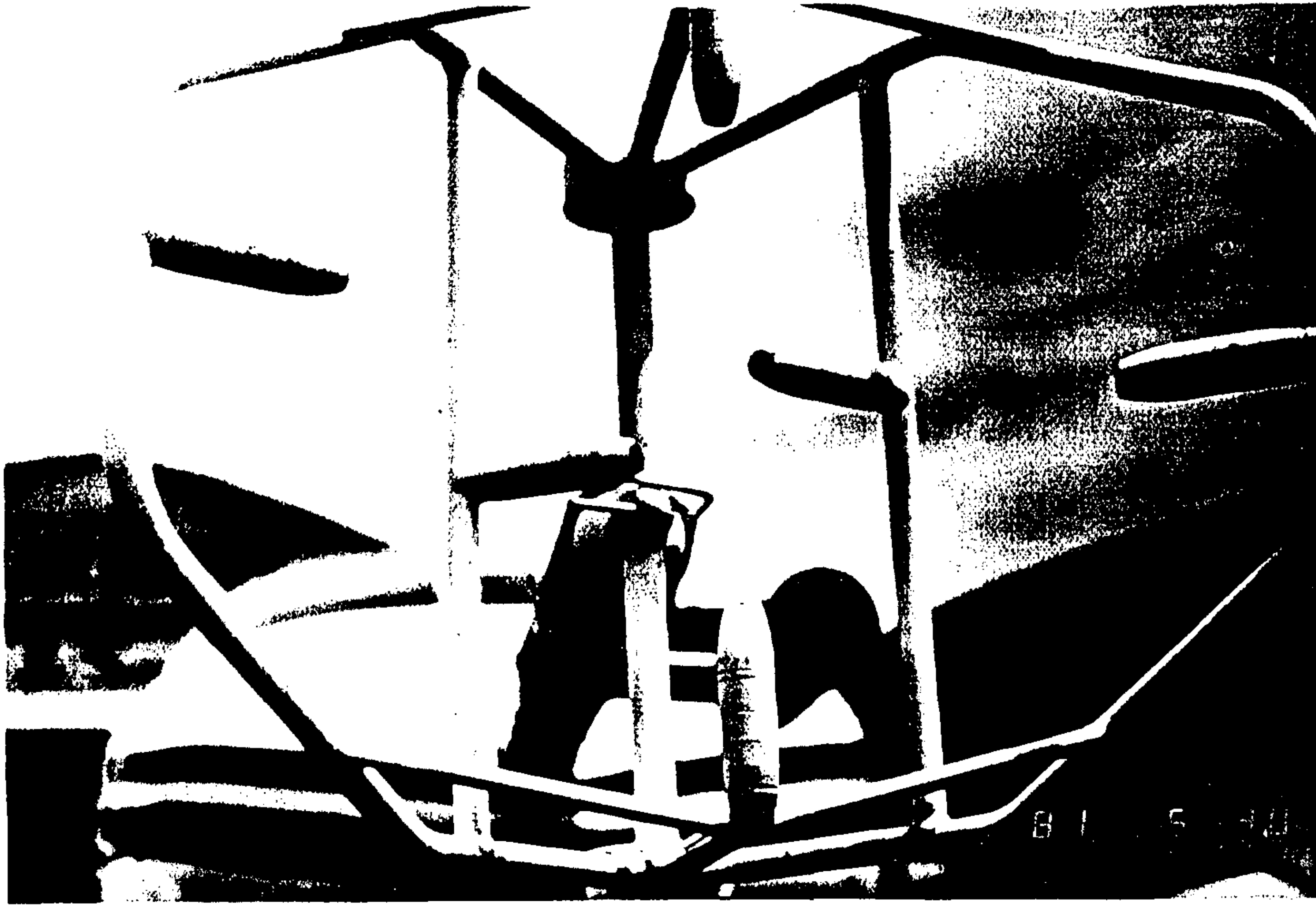
Temperature
- Sonic anemometer thermometer
- Platinum-wire resistance thermometer
- Thermo couple , Quartz thermometer

Water vapor
- Lyman-α hygrometer

CO_2, O_3, (CH_3)
- Ultra-red absorption

Velocity = (Prop. time I − Prop. time II) / 2L ∵ Prop. velocity of sound = Sound vel. in still air + Wind vel.

Temperature ∝ (Prop. time I + Prop. time II) / 2L



Averaging time $\mathcal{T}$ (Wyngaard 1973)

$$\mathcal{T} \approx 2\tau_i \; \overline{f'^2} \,/\, a^2 \bar{f}^2 \qquad \text{Lumley \& Panofsky (1964)}$$

$\mathcal{T}$: averaging time required to determine the mean f to an accuracy a

$\overline{f'^2}$: variance of f about its mean

τ_i : integral scale of time function $f \approx \frac{\Lambda}{U} \propto \frac{z}{U}$

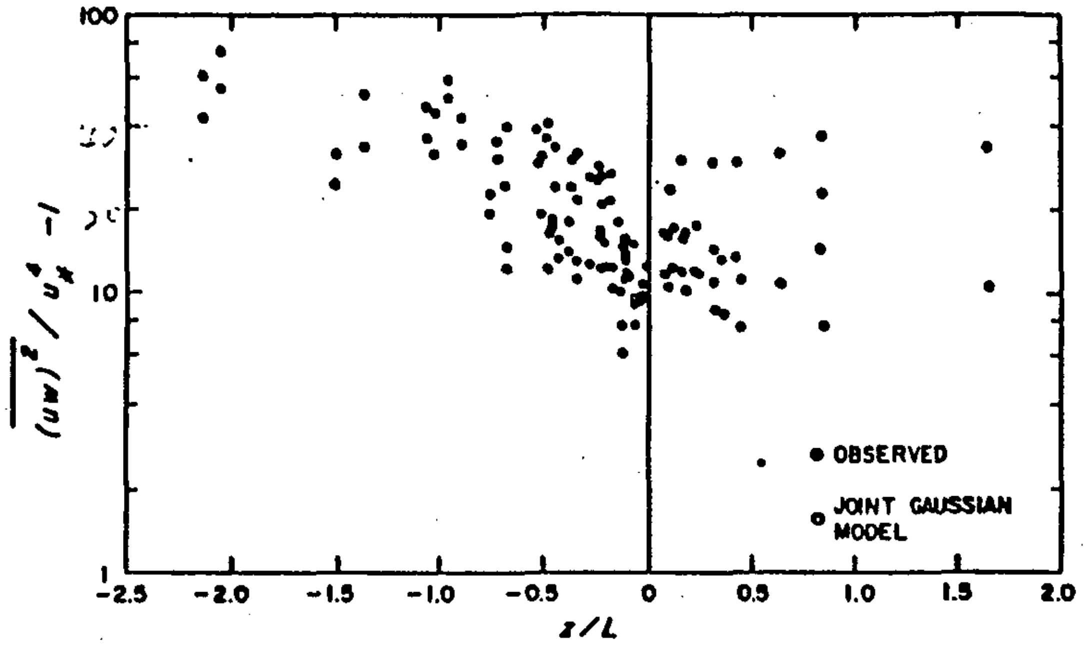
$$\mathcal{T}_{\overline{uw}} \approx \frac{2z}{a^2 U}\left[\frac{\overline{(uw)^2} - (\overline{uw})^2}{(\overline{uw})^2}\right] = \frac{z}{a^2 U}\left[\frac{\overline{(uw)^2}}{u_*^4} - 1\right]$$

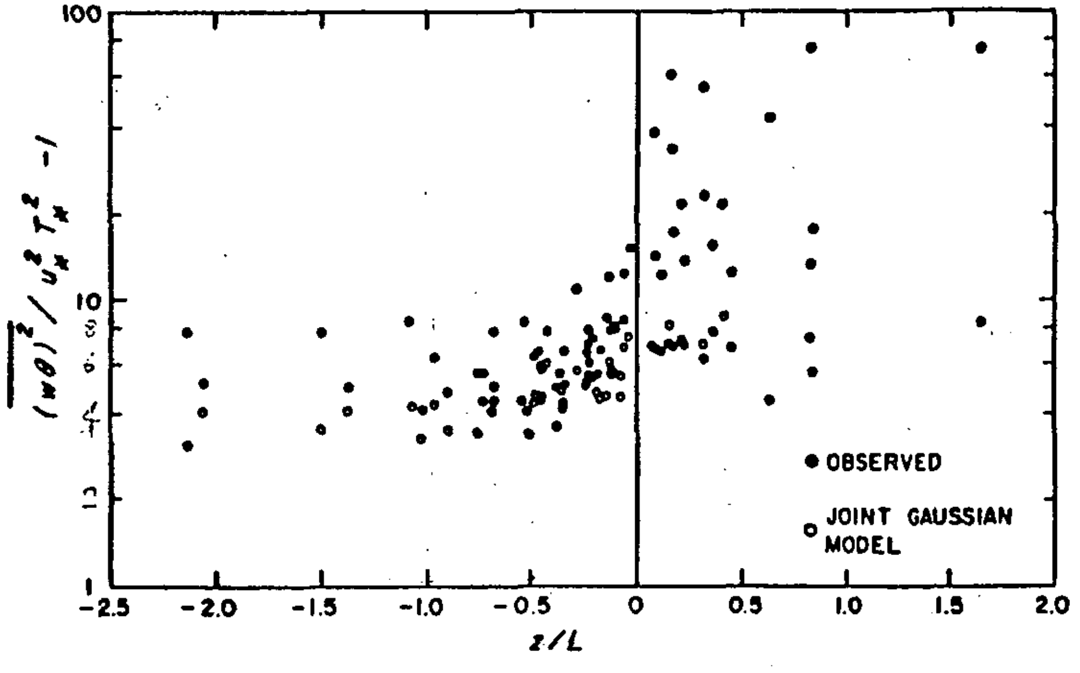
$$\mathcal{T}_{\overline{wc}} \approx \frac{2z}{a^2 U}\left[\frac{\overline{(wc)^2}}{u_*^2\, c_*^2} - 1\right]$$

$$\mathcal{T}_{\overline{w^2}} \approx \frac{2z}{a^2 U}\left[\frac{\overline{w^4}}{(\overline{w^2})^2} - 1\right] \qquad \left(\text{Flatness factor } F_w = \frac{\overline{w^4}}{(\overline{w^2})^2} \sim 3\right)$$

Gaussian.

Neutral	Kansas exp. (tower : 20m)
$\mathcal{T}_{\overline{w^2},\,\overline{c^2}} \approx 4\,\frac{z}{a^2 U}$	2.4 min
$\mathcal{T}_{\overline{wc},\,\overline{uw}} \approx 20\,\frac{z}{a^2 U}$	12 min
Unstable ($z/L = -1$)	
$\mathcal{T}_{\overline{w^2},\,\overline{c^2}} \approx 4\,\frac{z}{a^2 U}$	2.4 min
$\mathcal{T}_{\overline{uw}} \approx 100\,\frac{z}{a^2 U}$	60 min
$\mathcal{T}_{\overline{wc}} \approx 12\,\frac{z}{a^2 U}$	7.2 min


100
10
1
−2.5
−2.0
−1.5
−1.0
−0.5
0
0.5
1.0
1.5
2.0
z/L
$\overline{(uw)^2}/u_*^4 - 1$
OBSERVED
JOINT GAUSSIAN MODEL


100
10
1
−2.5
−2.0
−1.5
−1.0
−0.5
0
0.5
1.0
1.5
2.0
z/L
$\overline{(w\theta)^2}/u_*^2 T_*^2 - 1$
OBSERVED
JOINT GAUSSIAN MODEL

Indirect flux measurements

Concentration variance dissipation rate χ_c

(destruction rate of concentration fluctuation by molecular diffusion)

$$\chi_c = 6\mathcal{D} < (\frac{\partial c}{\partial x})^2 >$$

Budget equation for scalar variance

$$-2 <wc> \frac{dC}{dz} - \underbrace{\frac{\partial}{\partial z} <wc^2>}_{\text{negligible}} = \chi_c$$

Determination of χ_c

1. $\chi_c \equiv 6\mathcal{D} < (\frac{\partial c}{\partial x})^2 > = \frac{6\mathcal{D}}{U^2} < (\frac{\partial c}{\partial t})^2 >$ (frozen turbulence)

2. One dimensional spectrum

$$\varphi_{cc}(k_1) = \beta_c \chi_c \varepsilon^{-1/3} k_1^{-5/3}, \quad <c^2> = \int_0^\infty \varphi_{cc}(k_1)\,dk_1$$

$$\beta_c = 0.40$$

3. Structure function

$$<[c(x+r) - c(x)]^2> = C_d r^{2/3}$$

In the surface layer (Monin-Obukhov similarity theory)

$$\chi_c = \frac{u_* C_*}{\kappa z}\phi_c \qquad \varepsilon = \frac{u_*^2}{\kappa z}\phi_\varepsilon, \quad \chi_\theta = \frac{u_* T_*}{\kappa z}\phi_\theta$$

$\phi_\varepsilon, \phi_\theta, \phi_c$: universal function of z/L

Above the surface layer

Measurements of χ_c and $\frac{dC}{dz}$

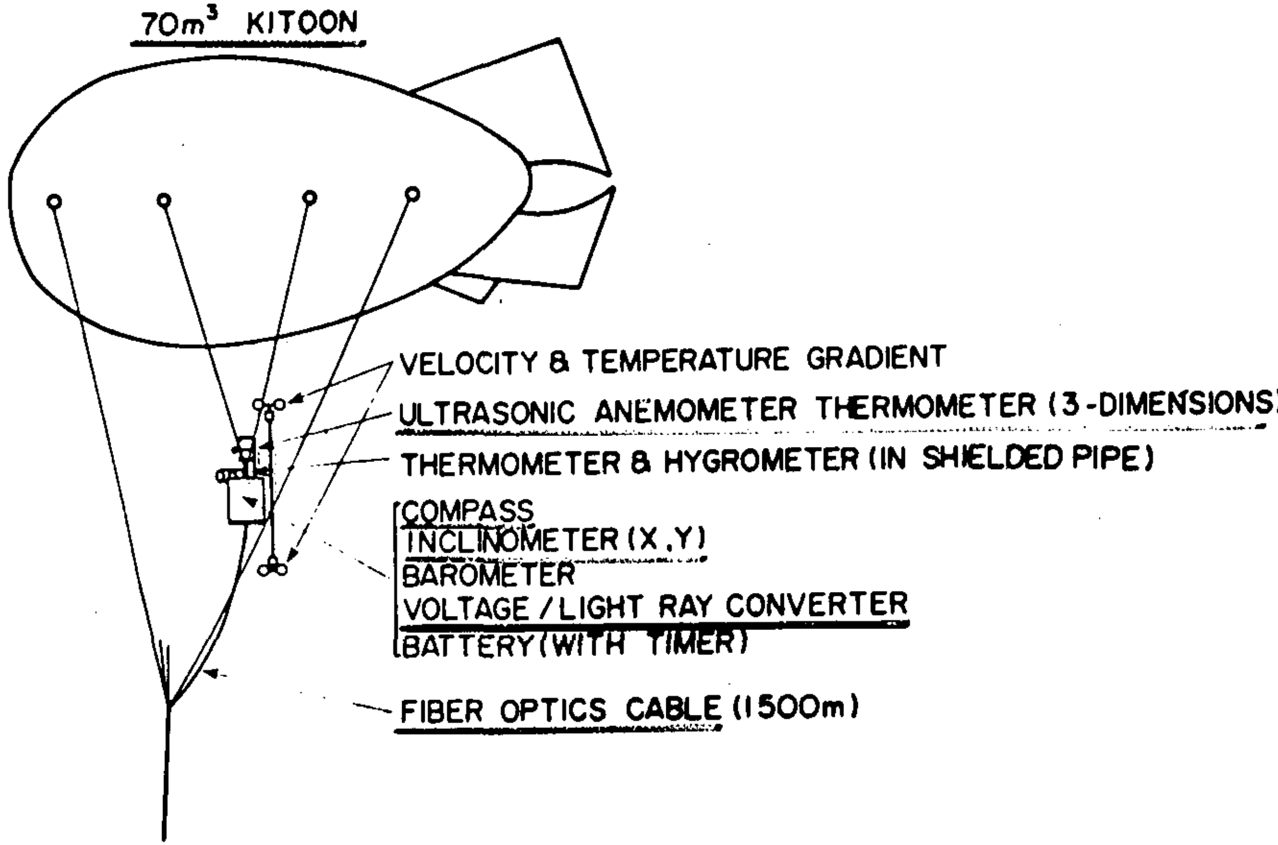
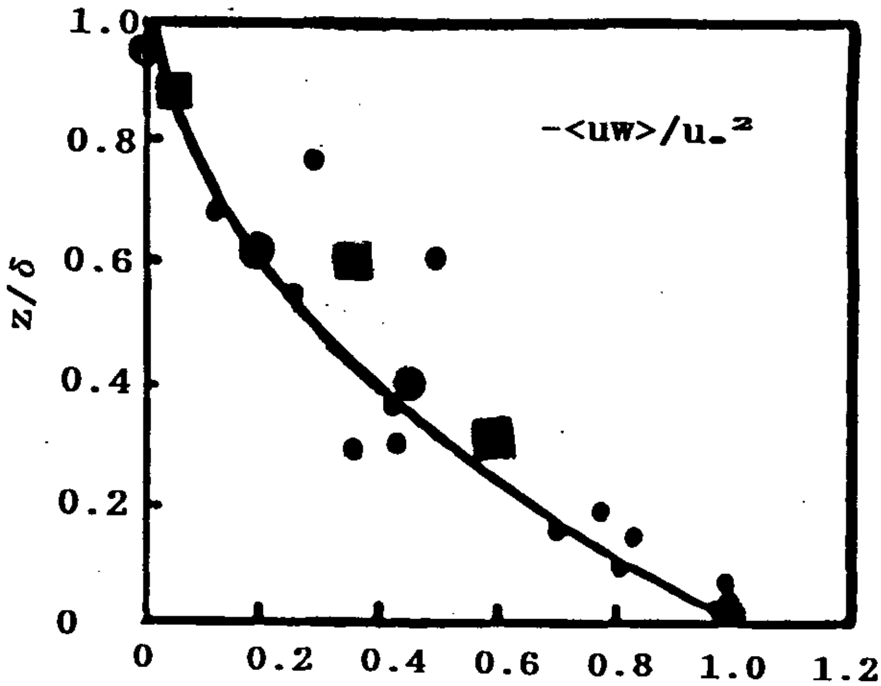
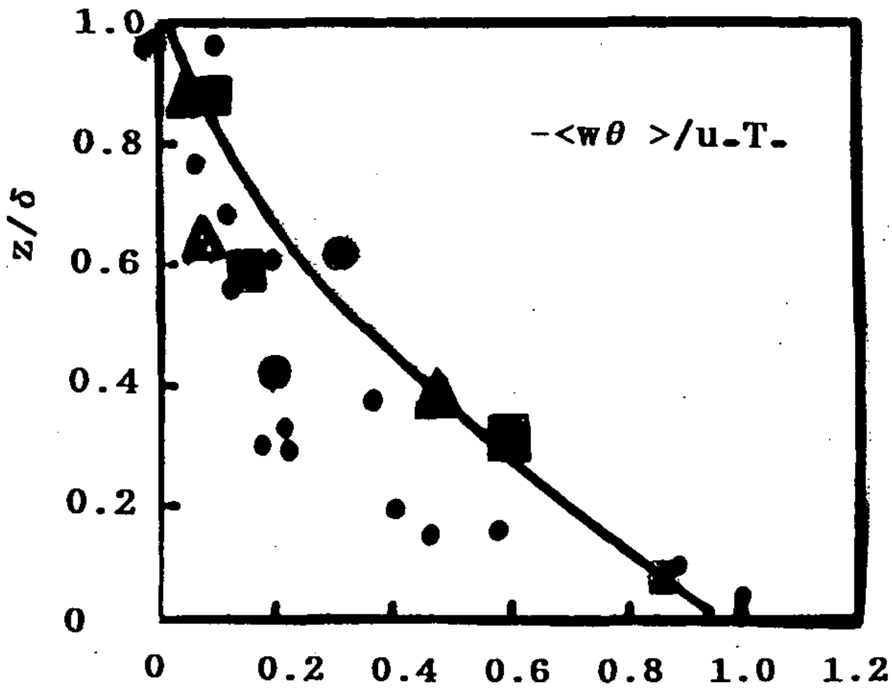


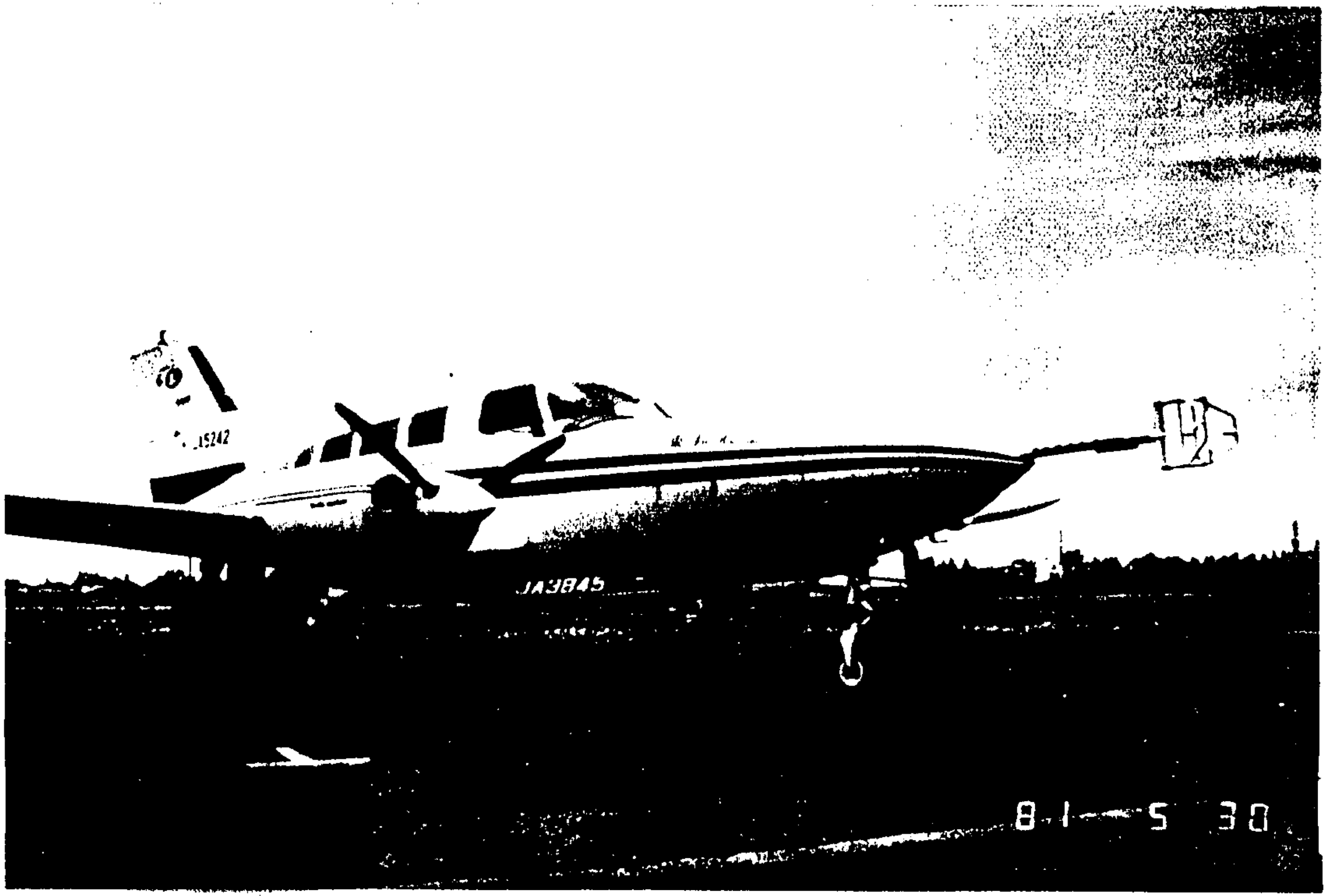
Fig. 1. Schematic of the system.



-<uw>/u.²
z/δ
1.0
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1.0
1.2

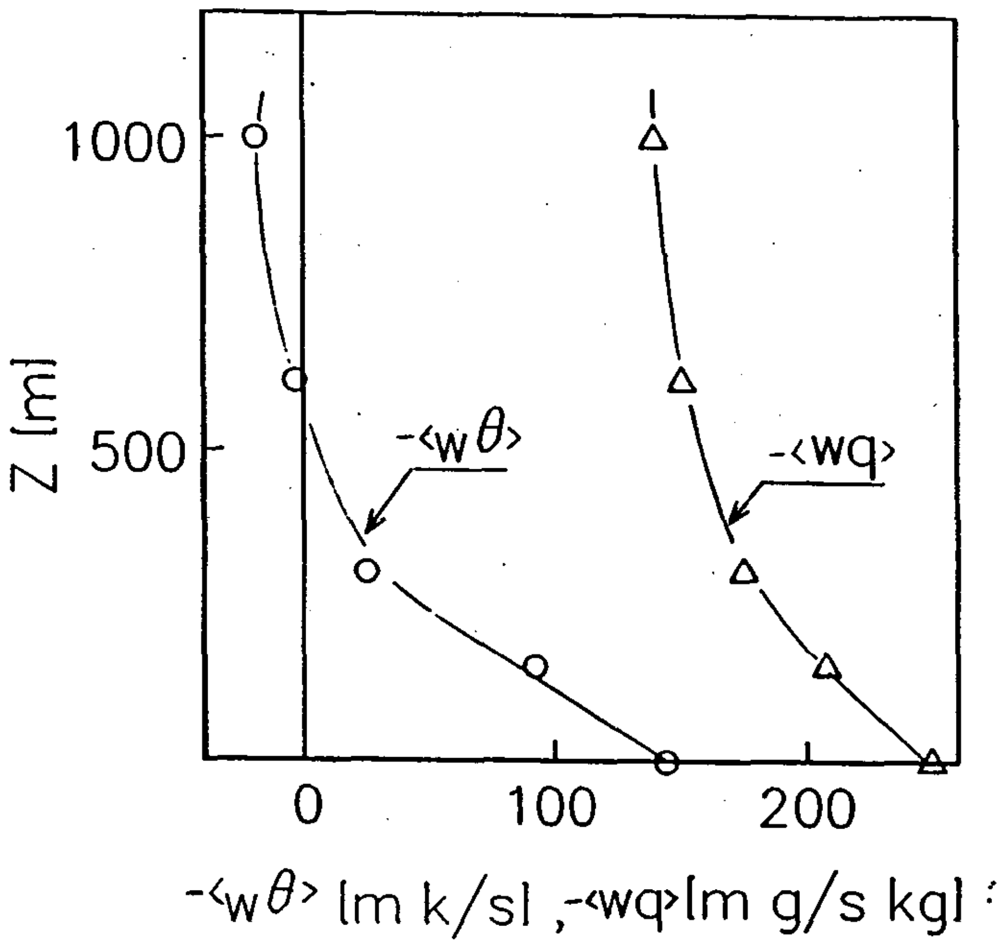


-<wθ >/u.T.
z/δ
1.0
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1.0
1.2


JA3845

Wind velocity against earth = Air velocity against plane SAT − Airplane velocity against earth INS → u W-E comp. v N-S comp. w Vertical comp.

Channel Contents	Measuring Device	Digital/Analog	Resolution(Nominal)
1 Longitude			
2 Latitude			$45° \times 2^{-19}$
3 Heading Angle			(0.309″)
4 Track Angle			(≈9.5 m for Longitude)
5 Ground Speed	INS	Digital	
6 N-S Velocity			0.1×2^{-6} KNT
7 E-W Velocity			(≈0.08 cm/s)
8 Pitch			0.02°
9 Roll			
10 Vertical Accel.			0.613 cm/s²
11 X			
12 Y-Yshift			0.01 m/s
13 Z	SAT	Analog	
14 Temperature			0.04° C
15 Altitude	Barometer		0.1 mb
16 Altitude	Radio-altimeter		0.66 m
17 Temperature	Quartz Thermometer		1/400° C
18 Ground/Sea Surface Temperature	IRT		0.01° C
⋮ ⋮	⋮	⋮	



1000
500
Z [m]
-⟨wθ⟩
-⟨wq⟩
0
100
200
-⟨wθ⟩ [m k/s] ,-⟨wq⟩[m g/s kg]

GAS FLUX MEASUREMENTS BY EDDY CORRELATION

M.R. Raupach
CSIRO Centre for Environmental Mechanics, Canberra, Australia

This talk describes some of the issues and problems involved in the field measurement of vertical scalar fluxes, especially trace gas fluxes, by the eddy correlation method. The talk is divided into 6 sections.

1. **Introduction and principle** (overheads 1 to 2): Recalling the basic principle of the eddy correlation method, that a vertical scalar flux F_C in a turbulent flow (with negligible molecular transport compared with turbulent transport) is given by $F_C = \overline{w\rho_C} = \overline{w}\ \overline{\rho_C} + \overline{w'\rho_C'}$ (where w is the vertical velocity, ρ_C the scalar concentration, overbars denote averages and primes fluctuations).

2. **Bandwidth** (overhead 3): Specifying the frequency range contributing to the eddy covariance.

3. **Webb-Pearman-Leuning (WPL) effect** (overheads 4, 4a): Correcting for the fact that $\overline{w}$ is nonzero over flat ground in thermally non-neutral conditions.

4. **Sampling through a tube** (overheads 5 to 9): Correcting for the damping of scalar fluctuations when air must be drawn through a tube to sample the scalar concentration.

5. **Corrections for platform motion** (overheads 10 to 12): Correcting for ship or aircraft motion.

6. **Avoiding eddy correlation - inverse methods for inferring fluxes from concentrations** (overheads 13 to 15): Introducing two new methods for inferring trace gas fluxes from concentration measurements: an inverse Lagrangian method for deducing source-sink profiles in plant canopies from concentration profiles and turbulence information, and a convective boundary layer (CBL) budget method for inferring trace gas fluxes at regional (100 km^2) scales.

EDDY CORRELATION

1. Principle : $F_c = \overline{w \rho_c} = \bar{w}\bar{\rho}_c + \overline{w' \rho_c'}$

2. Bandwidth : $10^{-3} < n < 10$

$n = fz/\bar{u}$

f = frequency
z = height
$\bar{u}$ = mean wind speed

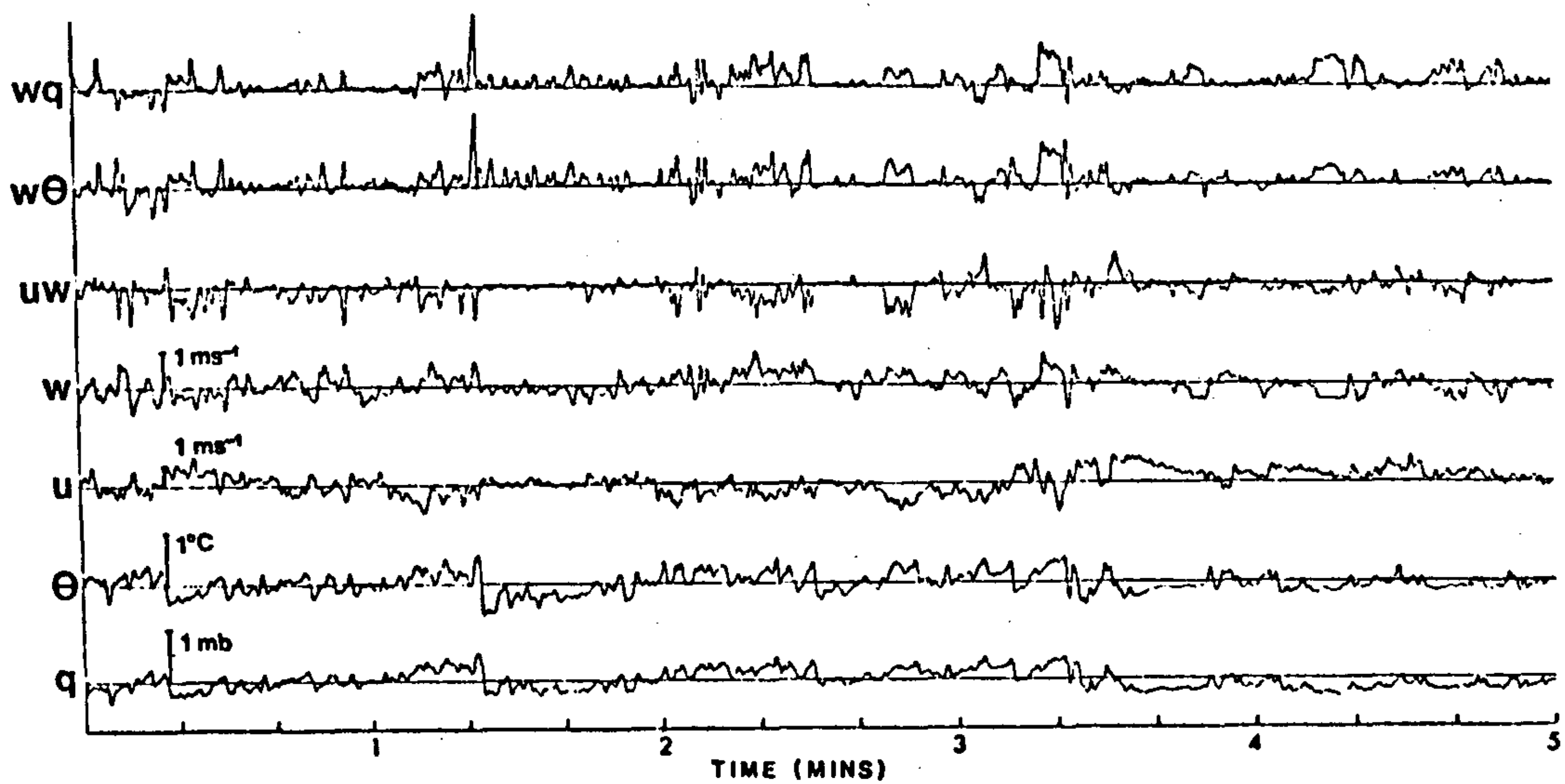
3. Webb-Pearman-Leuning effect

4. Sampling along a tube

5. Platform motion

6. Avoiding eddy correlation:
inverse methods for
inferring F_c from $\bar{\rho}_c$

LAKE ALBERT, SOUTH AUSTRALIA, APRIL 1975



LAKE ALBERT, SOUTH AUSTRALIA, 1975

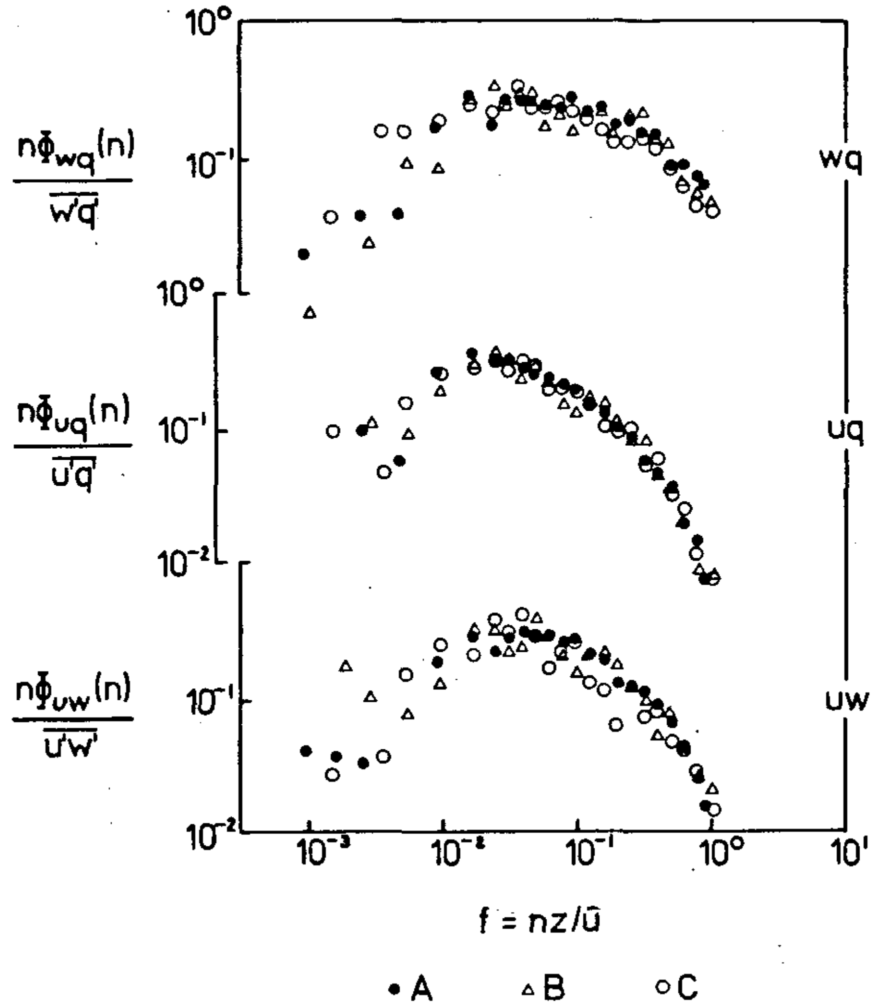
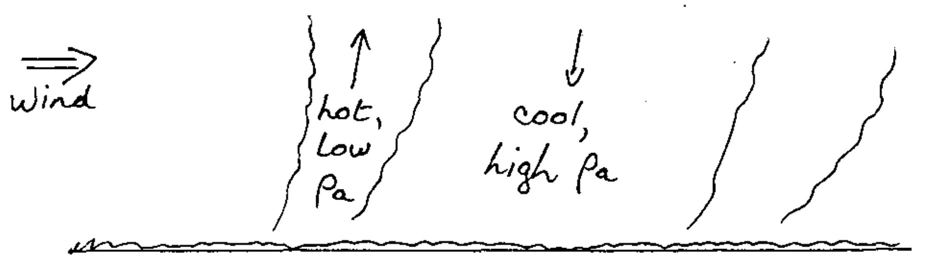


Figure 8. The wq, uq and uw cospectra. The data are grouped in stability bands thus:

band A: $0{\cdot}11 \leqslant -z/L < 0{\cdot}18$ (4 runs)
band B: $0{\cdot}18 \leqslant -z/L < 0{\cdot}21$ (3 runs)
band C: $0{\cdot}27 \leqslant -z/L < 0{\cdot}63$ (5 runs)

WEBB - PEARMAN - LEUNING (WPL) EFFECT



Vertical flux of dry air $= \overline{w\rho_a} = 0$

$$\Rightarrow \quad \bar{w}\bar{\rho}_a + \overline{w'\rho_a'} = 0$$

$$\Rightarrow \quad \bar{w} = -\,\overline{w'\rho_a'}/\bar{\rho}_a$$

$(\bar{w} > 0$ in convection$)$

For a trace scalar:

$$\underset{\text{(mass flux)}}{F_c} = \underset{\text{(density)}}{\overline{w\rho_c}} = \underset{\text{WPL correction}}{\bar{w}\bar{\rho}_c} + \overline{w'\rho_c'}$$

Open path sensor (with temp fluctuations):

$$\bar{w}\bar{\rho}_c = \bar{\rho}_c\left[\overline{w'm'} + (1+\bar{m})\frac{\overline{w'\theta'}}{\bar{T}}\right]$$

Closed path sensor (temp fluctuations removed):

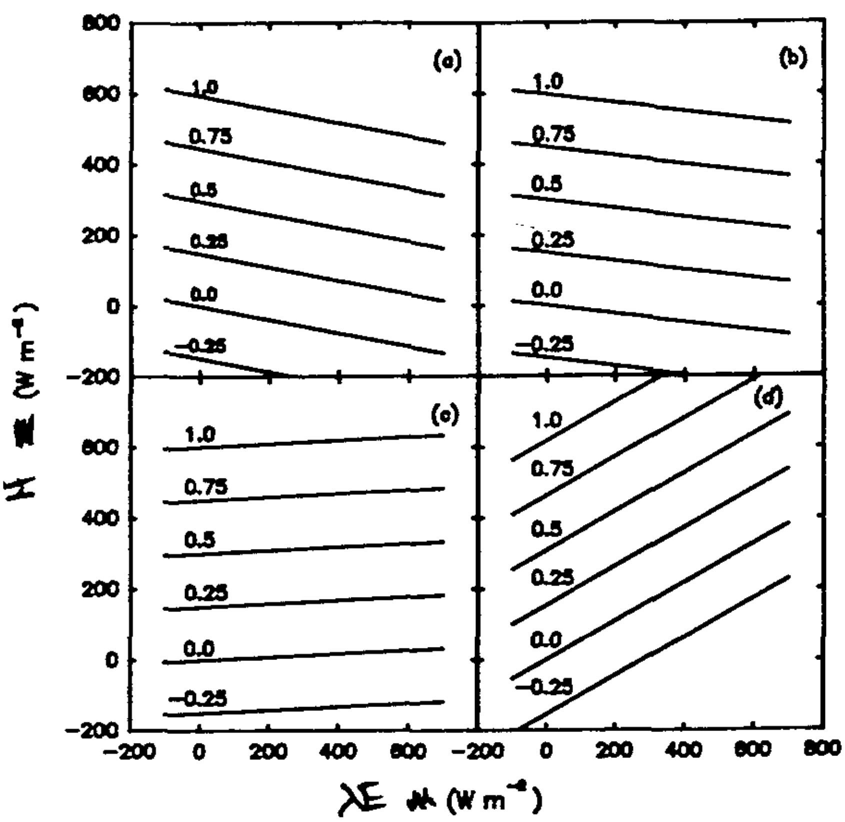
$$\bar{w}\bar{\rho}_c = \bar{\rho}_c\left[\overline{w'm'} + \bar{m}\frac{\overline{w'\theta'}}{\bar{T}}\right]$$

with $m = H_2O$ molar mixing ratio.

$\bar{w}\bar{\rho}_c/F_c$ range:

-0.1 to $\lesssim -1$		(CO_2)
0.25 to 10		(N_2O)
0.04 to 0.6		(CH_4)

LEUNING AND MONCRIEFF (1990)



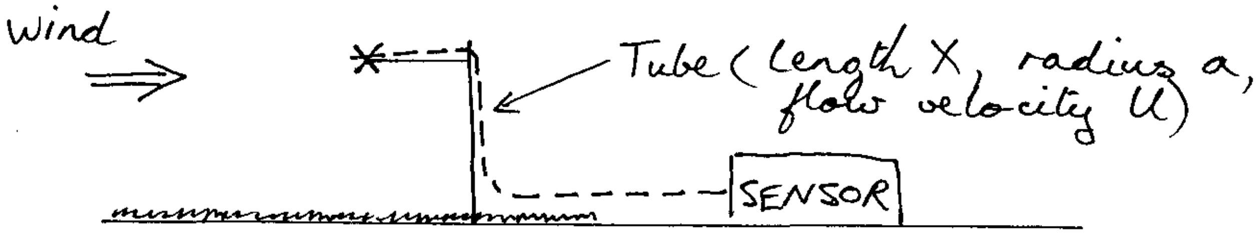
$\overline{w\rho_c}$ for CO_2 ($mg\ m^{-2}\ s^{-1}$)

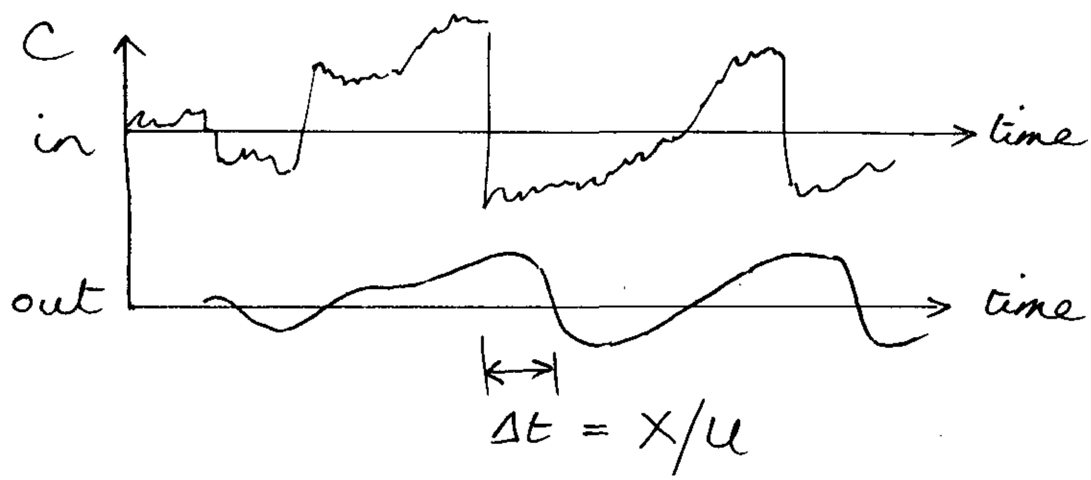
a → b → c → d represents increasing H_2O cross-sensitivity

Fig. 2. Isopleths for the correction to measured CO_2 fluxes as a function of latent and sensible heat fluxes for open-path CO_2 analysers with differing cross-sensitivity to water vapour. (a) $\beta/\alpha = 0$, (b) $\beta/\alpha = 3 \times 10^{-4}$, (c) $\beta/\alpha = 1 \times 10^{-3}$ and (d) $\beta/\alpha = 3 \times 10^{-3}$. Numbers on the lines represent the flux correction to be added to F_{raw} ($mg\ CO_2\ m^{-2}s^{-1}$). Corrections at $H = 0$ also apply to closed-path CO_2 analyser.

SAMPLING THROUGH A TUBE

Lenschow and Raupach, J. Geophys. Res. (1991) 96, 15259-68.





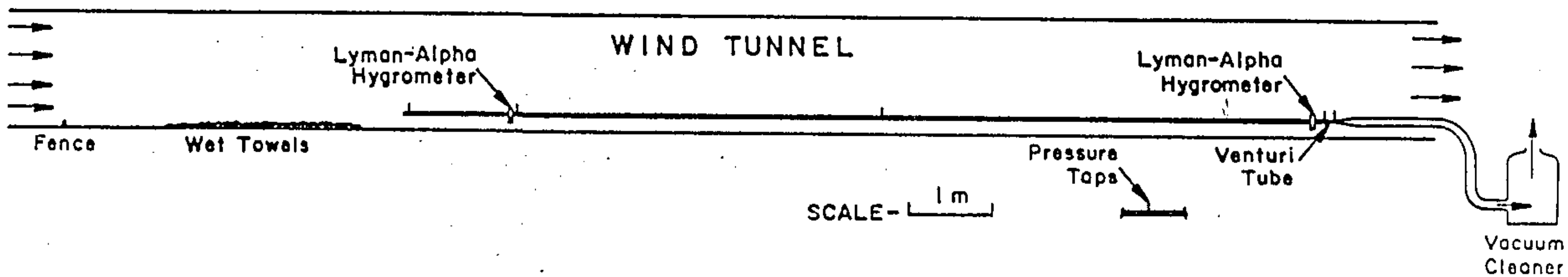
Tube transfer function:

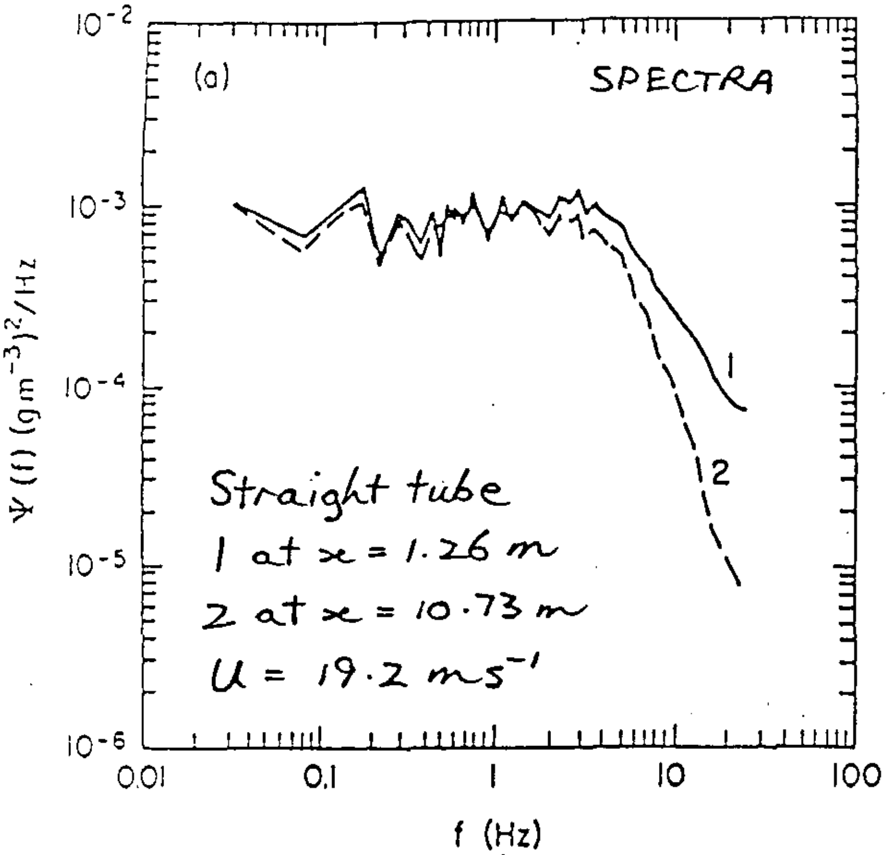
$$\Phi(n) = \frac{\text{amplitude out}}{\text{amplitude in}} = \exp\left(\frac{-n^2 \ln\sqrt{2}}{n_0^2}\right)$$

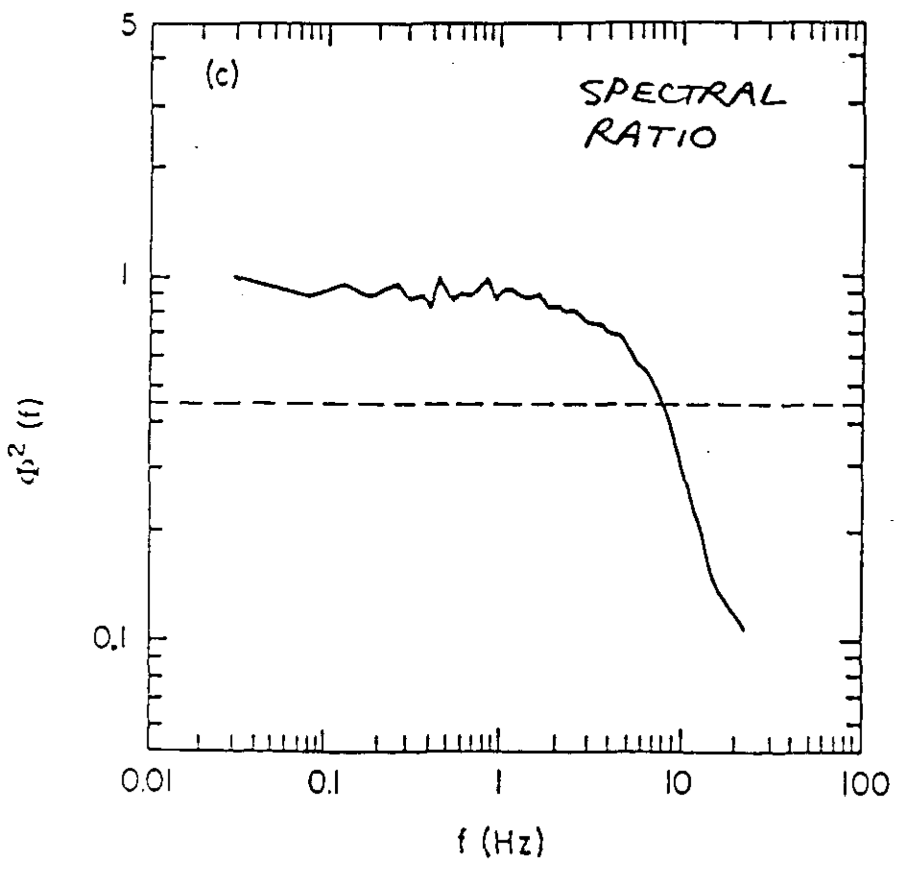
$$n = \frac{f}{U}\sqrt{aX} = \text{dimensionless frequency}$$

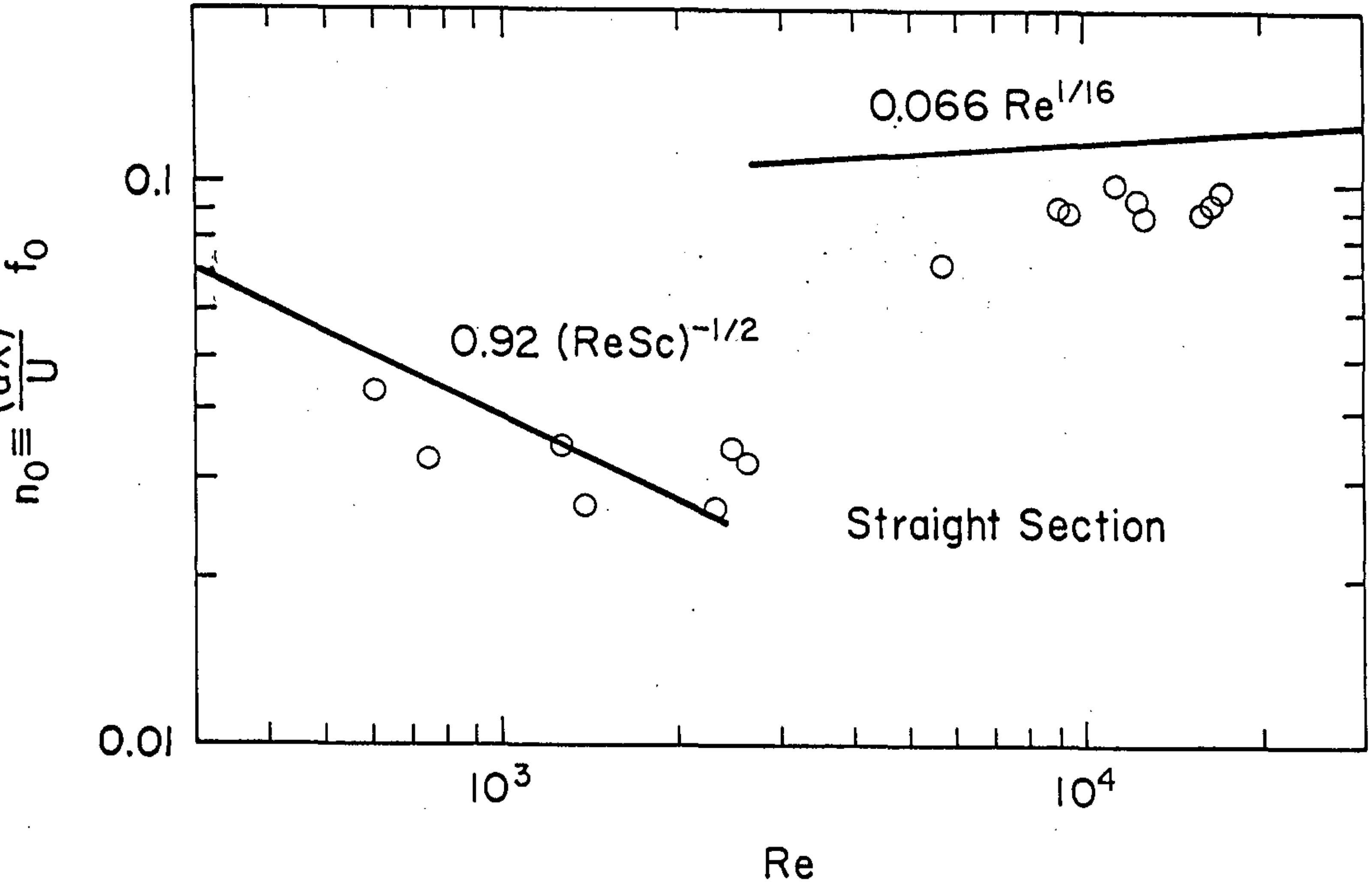
$$n_0 = \begin{cases} 0.92\,(Re\,Sc)^{-1/2} & \text{(laminar)} \\ 0.066\,Re^{1/16} & \text{(turbulent)} \end{cases}$$

Desirable: turbulent flow in tube! HIGH measurement point!


WIND TUNNEL
Lyman-Alpha Hygrometer
Lyman-Alpha Hygrometer
Fence
Wet Towels
Pressure Taps
Venturi Tube
SCALE- 1 m
Vacuum Cleaner


(a)
SPECTRA
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
Ψ (f) (g m^{-3})^2/Hz
1
2
Straight tube
1 at x = 1.26 m
2 at x = 10.73 m
U = 19.2 ms^{-1}
0.01
0.1
1
10
100
f (Hz)


(c)
SPECTRAL
RATIO
5
1
0.1
Φ^2 (f)
0.01
0.1
1
10
100
f (Hz)


0.066 Re^1/16
0.1
0.01
0.92 (ReSc)^-1/2
Straight Section
10^3
10^4
Re
n0 ≡ (aX)^1/2 / U f0

LEUNING AND MONCRIEFF (1990)

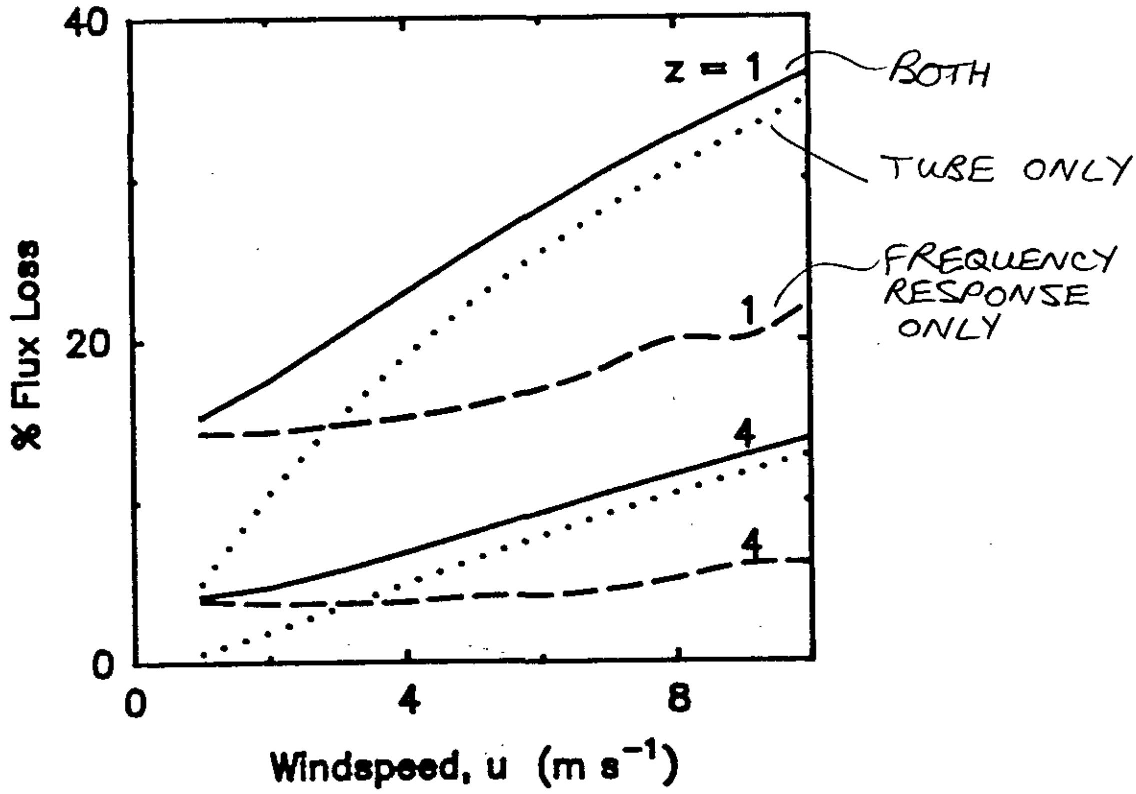
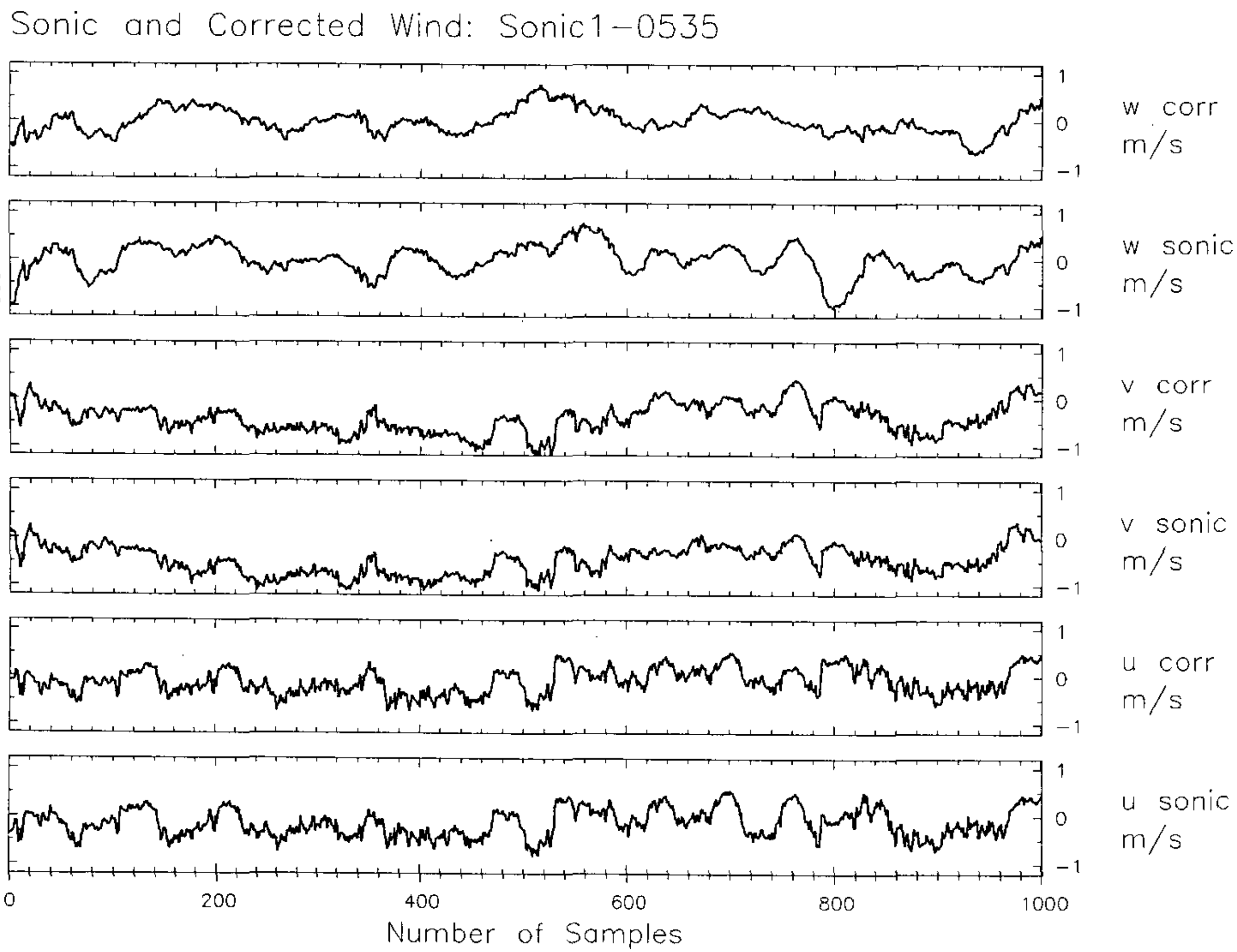


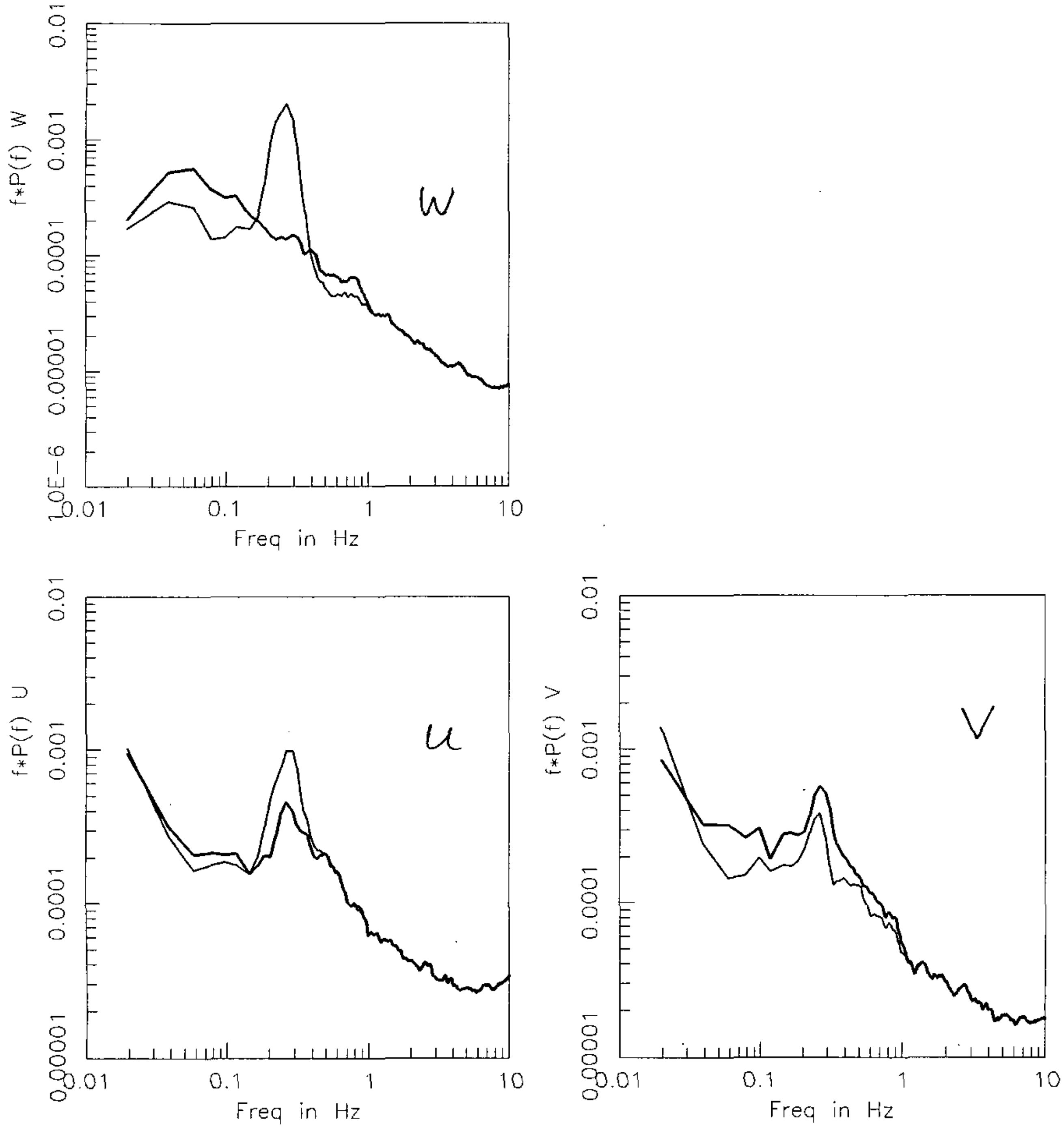
Fig. 3. Estimated percentage loss in measured F_{raw} for a closed-path CO_2 analyser supplied with air sampled through a copper tube $L = 1.5$ m, $r_0 = 3$ mm and $V = 8 \times 10\,m^{-3}\,s^{-1}$. Separation distance between tube inlet and sonic anemometer was 0.18 m. Losses are plotted against windspeed for sensor heights of 1 and 4 m above the zero-plane. Flux losses due to damping of turbulence fluctuations by tube only (......); flux losses due to limited sensor frequency response but neglecting effect of tube (----) and combined influence of tube and sensor frequency response (——).

Correction for ship motion:
Shao, Bradley, Coppin (current work)



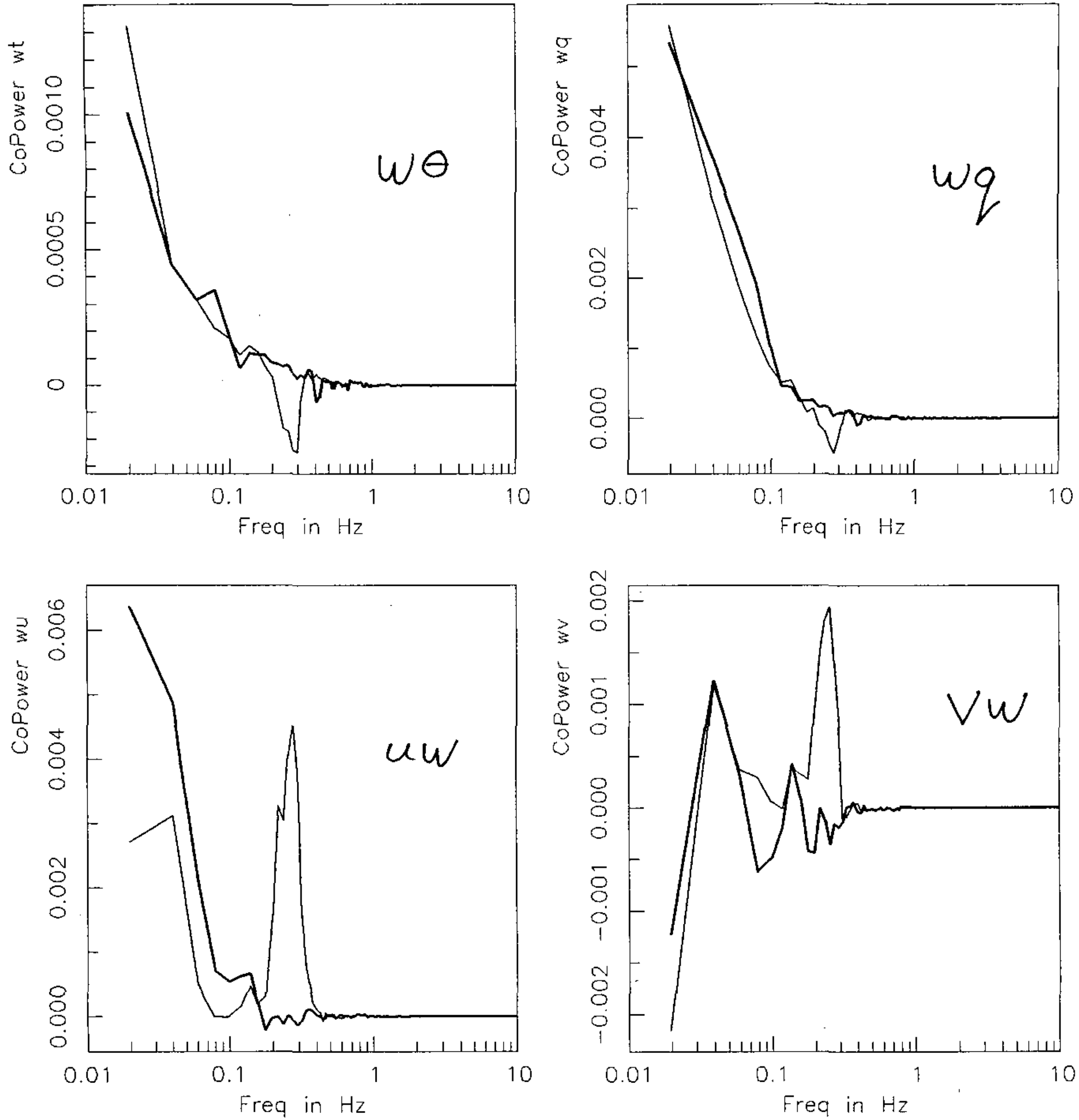
— 300 —

Sonic 1 - 0535



Spe

Sonic 1 - 0535



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INVERSE METHODS

– inferring fluxes from mean concentrations

1. Canopy scale

$$\underbrace{c_i - c_R}_{\substack{\text{concentration}\\ \text{at level } i}} = \sum_{j=1}^{m} \underbrace{D_{ij}}_{\substack{\text{dispersion matrix:}\\ \text{depends on } \sigma_w,\, T_L}} \underbrace{S_j \, \Delta z_j}_{\substack{\text{source strength}\\ \text{in layer } j}}$$

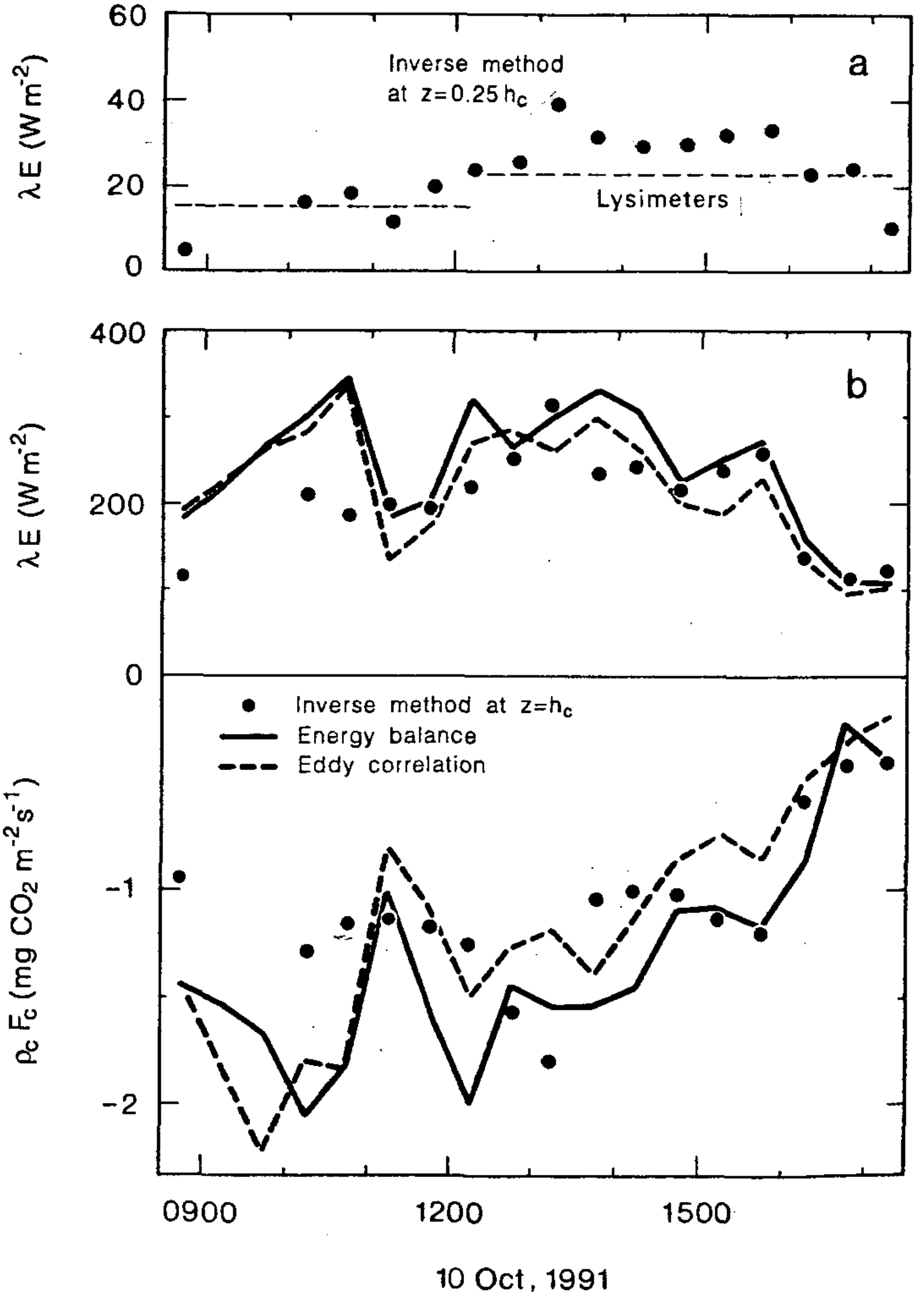
2. Scale of planetary convective B.L.

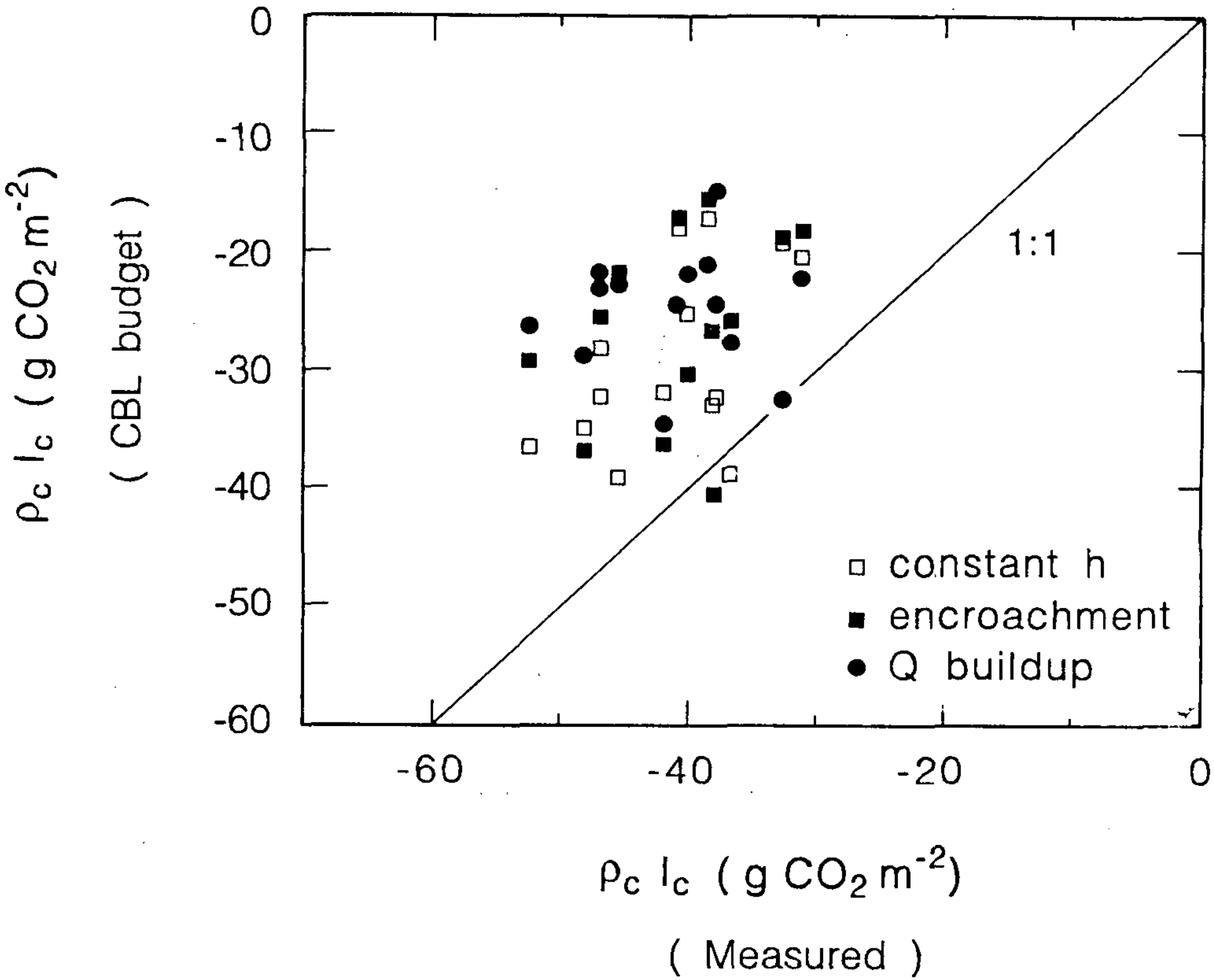
CBL well mixed

$$\frac{dC}{dt} = \frac{F_c}{h} + (C_+ - C)\frac{dh}{dt}$$

$$\Rightarrow \int_0^t F_c \, dt = (C - C_+)\, h(t)$$

(with assumptions)


60
40
20
0
λE (W m-2)
Inverse method
at z=0.25 hc
a
Lysimeters
400
200
0
-1
-2
λE (W m-2)
b
Inverse method at z=hc
Energy balance
Eddy correlation
ρc Fc (mg CO2 m-2 s-1)
0900
1200
1500
10 Oct, 1991


0
-10
-20
-30
-40
-50
-60
-60
-40
-20
0
1:1
constant h
encroachment
Q buildup
ρc lc (g CO2 m-2)
(CBL budget)
ρc lc (g CO2 m-2)
(Measured)