

## **From Impact to Emission : Tolerable Windows Approach**

**F. Toth**

## THE TOLERABLE WINDOWS APPROACH TO INTEGRATED ASSESSMENTS

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**Abstract:** The Tolerable Windows Approach is based on external normative specifications of tolerable sets of climate impacts as well as proposed quotas and policy instruments for implementation. The related "tolerable windows" (TWs) are derived successively in a "backwards mode". First, a tolerable window for future climate development is deduced by analyzing the maximum stress levels caused by climate change that one can assume to be ecologically and socioeconomically bearable. Next, the corresponding set of admissible emission profiles is calculated, i.e. global greenhouse gas emission paths which keep the climate system within the established window. Finally, these paths are evaluated against various criteria to derive optimal emission reduction policies.

The objective of the TWA is twofold: firstly, to determine complete sets of all admissible emission paths which are compatible with the normative inputs and, secondly, to select optimal emission paths in a "second best" manner. While the second task can be embedded in the usual cost-effectiveness framework, the first one requires an appropriate mathematical foundation which is based on the theory of differential inclusions. The methodological tools we developed appropriately take into account the dynamic aspects of the entire problem, e.g. restrictions on the rate of climate change or socioeconomic conditions. From this point of view, the TWA may be considered as a dynamic generalization of the critical load concept. The solution methods are suitable for providing necessary and sufficient descriptions for the complete set of all admissible emission paths, i.e. for all emission paths which are compatible with the predefined windows. The boundaries of the tube of all admissible paths (called a "funnel" in mathematical terms) are calculated for different climate windows and socioeconomic restrictions. The funnels are by no means "safe" corridors. Although all admissible paths lie within the corridor (=funnel), not every arbitrary path lying within the limits of the corridor is admissible. The required insight into the internal structure of the funnels is gained by implementing an appropriate parameterization of the emission profiles. Finally, the selection of cost-effective emission paths is conducted with the MERGE model in accordance with our climate windows.

### **1. Introduction: background, objectives**

The project on Integrated Assessment of Climate Protection Strategies (ICLIPS) is an international and interdisciplinary research activity seeking to provide new insights in the problem of global climate change by pursuing a new concept, the tolerable windows approach (TWA). The project brings together experts from leading research institutions in the field of global climate change under the leadership of the Potsdam Institute for Climate Impact Research (PIK). PIK initiated this project and will integrate the various model components within the framework of TWA. The main objective of the ICLIPS project is to develop methods, collect data, and elaborate models needed to support the international decision-making community in implementing the Framework Convention on Climate Change (FCCC) and the Berlin Mandate.

The TWA is based on external normative specifications of tolerable sets of climate impacts, as well as proposed quotas and policy instruments for implementation. The associated "tolerable windows" (TWs) are derived successively in a "backwards mode". First, a tolerable window for future climate development is deduced by analyzing the maximum stress levels caused by climate change that one can assume to be ecologically and socioeconomically bearable. Next, the corresponding set of admissible emission profiles is calculated, i.e. global greenhouse gas emission paths which keep the climate system within the established window. Finally, these paths are evaluated against various criteria to derive optimal emission reduction policies.

This paper presents the first results of the project. Section 2 provides an overview of the inverse approach. This is followed by selected examples of climate response functions for determining acceptable levels of climate change. A concise presentation of the climate module is provided in Section 4 followed by a summary of results and lessons we learned from the first experiments. The last section summarizes the major points and main directions for future efforts.

It is important to note at the outset that most results presented here are very preliminary. Some parts contain dummy data in order to illustrate the basic concept (e.g., response surfaces). The climate module is based on a series of experiments with various versions of the pulse-response model (Meier-Reimer and Hasselmann, 1987). Finally, the MERGE model (Manne et al (1995)) was used to conduct the first experiments in integrating the climate window with an economic model.

## 2. The inverse approach: an overview

In the ideal situation when we possess full information about all relevant aspects of an environmental problem, economists would use cost-benefit analysis (CBA) to establish the optimal level of intervention. The climate change issue is no exception and integrated cost-benefit assessments have indeed been used to provide insights for policy. Unfortunately, there are some major weaknesses associated with at least some parts in these IAMs. These limitations characterize particularly non-market damages. A large number of studies have shown that serious threats, the most compelling reasons why we should worry about climate change at all, are expected to come from outside the national accounts. This suggests that there might be room for other approaches until we get improved inputs about the benefit components of a cost-benefit framework. This by no means implies that CBA should be abandoned. On the contrary, it is argued that the kind of analysis presented in this paper will also provide useful information for CBAs.

The convenient and traditional direction for IAMs has been to assume a baseline scenario of socioeconomic development, the associated business-as-usual greenhouse-gas emissions, and then work all the way through atmospheric concentrations of greenhouse gases, to the induced atmospheric forcing and global climate change, to regional patterns of climate change and biophysical impacts, and to economic implications. This framework has provided the necessary information for the various cost-benefit studies conducted so far.

A number of studies have been devised recently that follow an inverse path. The paper by Richels and Edmonds (1995) was one of the first attempts to formulate an inverse analysis. They defined hypothetical upper limits for atmospheric CO<sub>2</sub> concentration and searched for cost minimizing paths to reach those concentration levels over the long term. This idea has been developed further in Wigley *et al.* (1996). The German Advisory Board on Global Change (WBGU) defined a tolerable climate window based on criteria derived from the Earth's geological history and from other considerations (WBGU, 1995). From these perceived climate constraints, the assessment works backwards and seeks to find optimal emission paths that are

acceptable from the perspectives of various socioeconomic considerations. Yet another example of this backward analysis is the "Safe emission corridor" project conducted by the National Institute for Public Health and the Environment (RIVM) in the Netherlands (Alcamo and Kreileman, 1996).

All these studies need some basic understanding of the responses of various ecological and socioeconomic systems to the external forcing imposed by climate change. The approach followed as a necessity in CBA implies assigning monetary values to each of those exposure units (or valued environmental components) in order to measure the losses inflicted by climate change with a common yardstick. In contrast, the approach presented here seeks to define response functions in physical units.

Figure 1 presents the conceptual framework for our inverse approach. Response functions depict changes in valued environmental components (exposure units or impacts sectors) induced by climate change. These functions synthesize the best available information from impact assessment studies. The TWA analysis starts by defining tolerable impacts in various regions and impact sectors of concern. Resulting constraints on changes in various regional or global climate attributes serve as inputs to simple GHG emission/climate change models, to calculate sets of admissible emission paths that keep atmospheric concentrations of GHGs and resulting climate change within the permitted range. The final phase of this process adopts various decision criteria and relevant normative decisions from the policy arena, to identify optimal emission paths according to those criteria. We have learned in the first experiments that not each step delineated in Figure 1 can be separated and it is easier not to express and perform all relations as inverse calculations.

TWs may consist of discrete choices or connected sets of continuous variables concerning the magnitude or the rate of change of environmental, political and/or socioeconomic conditions. From the mathematical point of view, TWs impose restrictions not only on state variables  $\mathbf{x}(t) \in \mathbf{D}(t)$  (like global mean temperature) and control variables  $\mathbf{u}(t) \in \mathbf{U}(\mathbf{x}(t), t)$  (like global greenhouse gas emissions), but also on rates of change in state variables  $\dot{\mathbf{x}}(t) \in \mathbf{F}(\mathbf{x}(t), t)$  (e.g., rate of temperature change), where the vectors  $\mathbf{D}$ ,  $\mathbf{U}$  and  $\mathbf{F}$  are appropriately chosen set-valued functions of the indicated variables. Constraints of the latter type, i.e., those involving time derivatives of the state variables, are called differential inclusions. They reflect, for example, that climate damage may depend on temperature as well as on the rate of temperature change (Tol, 1995) and that abatement costs are not only a function of the total emission reduction but also, due to economic inertia, of the emission reduction rate (Grubb et al., 1995).

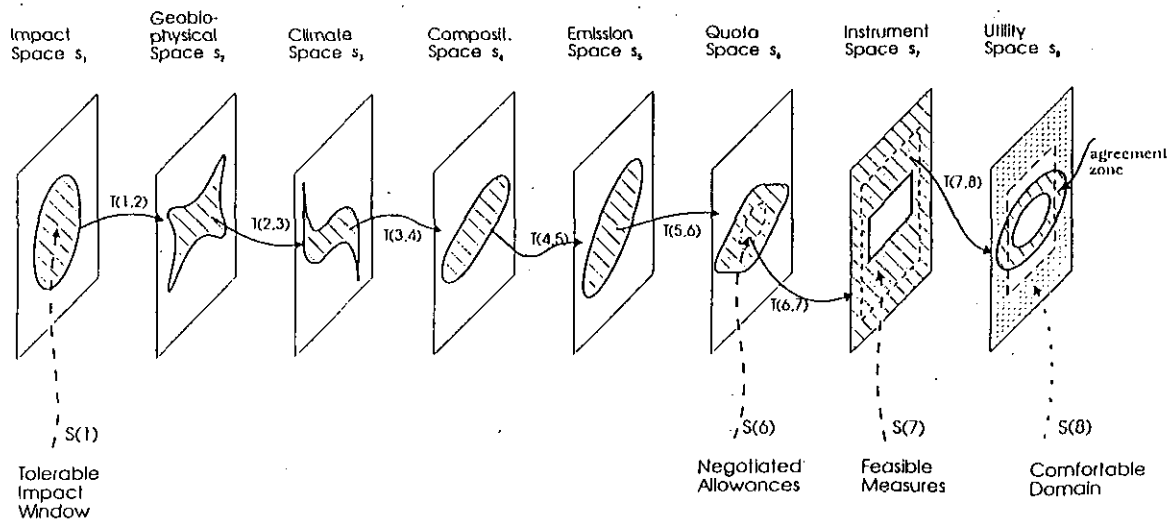
A mathematically sound treatment of dynamical systems including differential inclusions is an integral part of the "Theory of Differential Inclusions" (Deimling, 1992; Aubin and Cellina, 1984). This yet evolving theory provides appropriate definitions, a consistent theoretical background (e.g. theorems about the existence of solutions), and some useful approximate solution methods. Since we are interested not only in a single (along some criterion optimal) policy path but in the totality of all admissible policy paths, the question arises what will be considered a "solution" of the problem in this unfamiliar situation of non-uniqueness. At the current stage of the theory, the unabridged and exact determination of the totality of all admissible paths is hardly tractable. Even if it were possible, the solution would have to be described in a very abstract function space, so that, at least for non-mathematicians, this kind of "solution" must be translated into comprehensible partial solutions describing special features of the most comprehensive solution.

The solution strategy focuses directly on the determination of such partial solutions which in general may be considered as projections of the most comprehensive solution onto

appropriately defined subspaces. From the mathematical point of view, calculating combined TWs (i.e., those subsets of initially defined TWs which are consistent with all restrictions imposed upon by the other TWs) and determining corridors, which are called “funnels” (i.e., tubes of all admissible trajectories), are the most interesting issues. These funnels represent necessary conditions for the admissible time evolution of pertinent variables (e.g., emissions). They can be determined either by numerical solution of the integral funnel equation (Panasyuk, 1990) or by successive dynamic minimizing and maximizing of the relevant variable (e.g., emission) at several discrete points in time covering the whole time span considered (see Hofer and Tibken, 1994). Each scenario with a tolerable climate evolution lies well inside the different funnels. However, any point inside the funnels can be reached by at least one admissible trajectory, but the funnels are by no means “safe” corridors since, in general, not every arbitrary path lying inside the funnel is an admissible policy path. Therefore it is necessary to analyze the internal structure of these corridors.

A comprehensive method for the analysis adopts parameterized policy paths (e.g., emission profiles) involving only a few (e.g., 3) parameters and reflecting, to some extent, the shape of the corresponding funnel. By scanning relevant parts of the parameter space, all parameter combinations leading to admissible policy paths under the specified conditions can be determined. The resulting TW in the parameter space represents a sufficient condition for the totality of all admissible policy paths. Another method to “solve” the differential inclusion implies selecting a specific path, e.g., by optimization in a “second best” manner.

#### ICLIPS: Inverse Translation Strategy



**Figure 1: General structure of the inverse iteration procedure underlying the Tolerable Windows Approach.** Starting from predefined tolerable impact windows on the left, the corresponding admissible climate evolutions, emission profiles, and socioeconomic developments are derived by proceeding to the right. Additional normative input at different steps is indicated by dashed arrows.

### 3. Impact module: response functions for determining acceptable levels of climate change

This section presents a few illustrations derived from two different approaches to response functions. Both approaches take agricultural impacts and food supply as their climate “exposure units” (Parry et al., 1996). The two case studies were developed by external cooperation partners of the ICLIPS project (Parry and Livermore, 1997; Alcamo and Haupt, 1997) and proved to be very helpful in understanding the concept of response functions and sorting out relative merits and shortcomings of various approaches. It is crucial to emphasize that, in their current form, these examples are only precursory illustrations of possible ways to frame the notion of climate response functions. Hence the emphasis is on the concept rather than the actual numbers.

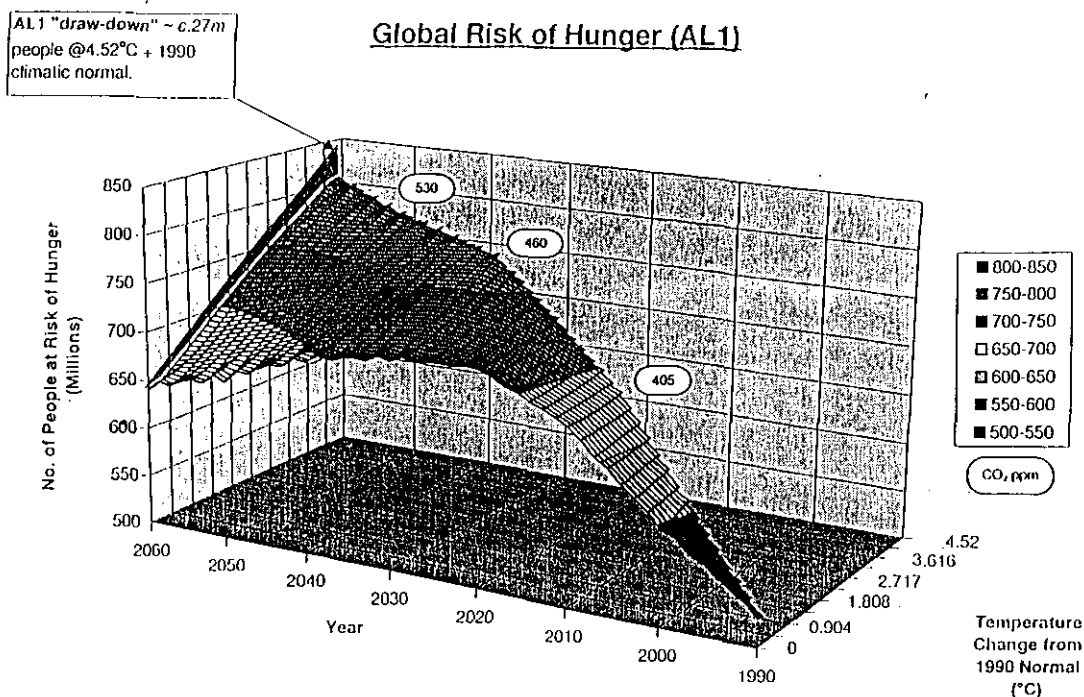
The first type of response surfaces, as shown in Figures 2 to 4, follows the ‘policy evaluation approach’ where a restricted number of scenarios is evaluated in detail (Parry and Livermore, 1997). The ‘risk of hunger’ indicator denotes the number of people “with an income insufficient to either produce or procure their food requirements”. This definition corresponds to the FAO methodology (FAO, 1987). The actual data have been derived by coupling: the GISS GCM; several IBSNAT crop models that include temperature and precipitation requirements as well as CO<sub>2</sub> fertilization; and the BLS (Basic Linked System), a general equilibrium model system with special emphasis on international food trade (Rosenzweig and Parry, 1994).

All calculation results are based on the same socioeconomic scenario, using the UN medium population estimates, assuming moderate economic growth, and taking into account trade liberalization and increasing productivity in agriculture. The ‘front end’ of each response surface in Figures 2 to 4 provides a ‘reference’ projection for the number of people at risk of hunger assuming *no* climate change, while the (diagonal) ‘back end’ refers to the climate change scenario calculated with the GISS GCM for an emission path that leads to a CO<sub>2</sub> doubling from pre-industrial levels by 2060. In this preliminary study, the interior of the surfaces has been derived by interpolation.

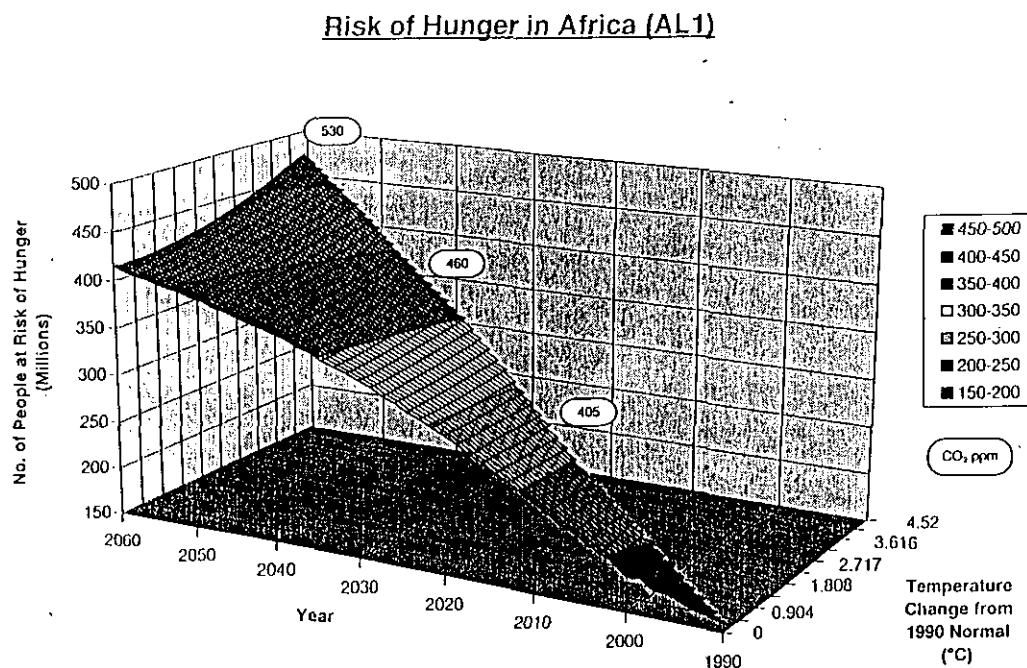
Figure 2 shows that even in the reference scenario, global risk of hunger is expected to increase from about 500 million in 1990 to about 650 million in 2060. For the climate change scenario, the latter value is as high as 810 million when the adaptation measures of AL1 (including shifts in planting date, additional irrigation, and changes in readily available crop variety) are taken into account. For the ‘no adaptation’ case, an additional 27 million people at risk of hunger is projected for the year 2060.

We can gain more insight in the underlying trends by looking at regional results for the two most interesting regions. In Africa (see Figure 3), climatic change is expected to have relatively small adverse effect compared to the tremendous increase in the number of people at risk of hunger projected already for the reference scenario, mainly due to sustained rapid population growth. The situation is completely different for South and South-East Asia (excluding China) (see Figure 4) where, after a temporary increase, the model projects an impressive decrease in risk of hunger despite continuing population growth. However, the adverse effect of climatic change is enormous in this region: with no adaptation measures employed about 220 million people are placed at risk of hunger in 2060, compared to 130 million if no climate change occurs.

The second type of response surfaces follows the ‘sensitivity approach’ (Alcamo and Haupt, 1997). Here, parts of the IMAGE 2 model (Alcamo, 1994) have been used to calculate potential crop yields under a range of possible climate states for two predominant crops in South and South-East Asia: rice (Figure 5) and pulses (Figure 6). The results suggest that both rice and pulses would be negatively affected by an increase in annual mean temperature and by a decrease in annual precipitation, with rice being more sensitive to a water shortage than pulses.

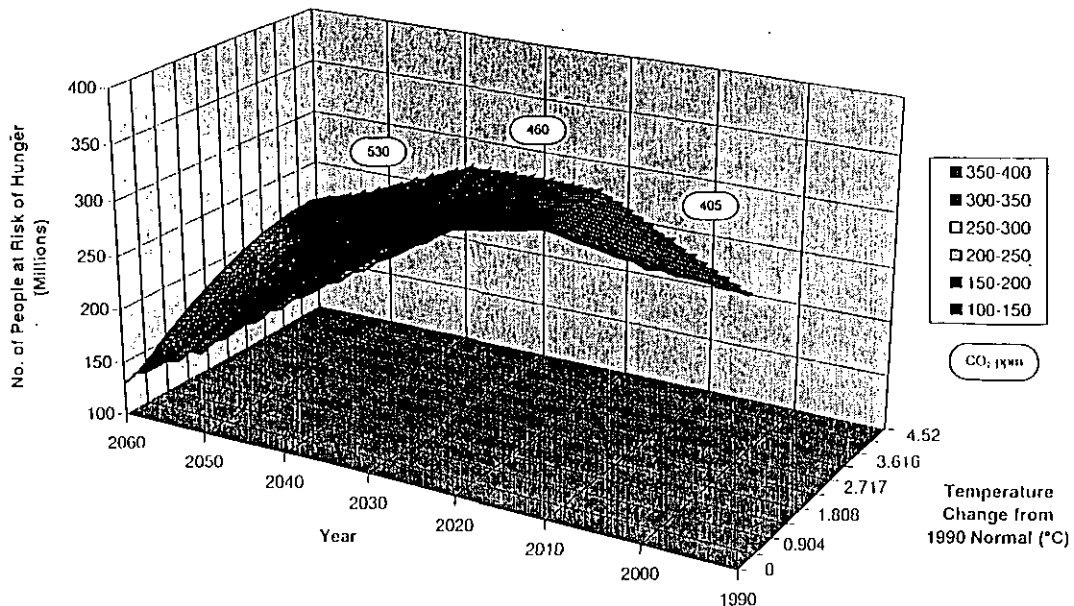


**Figure 2:** Number of people at risk of hunger in the world from 1990 until 2060 for a 'no climate change scenario' (front-end of the surface) and for a 'CO<sub>2</sub> doubling scenario with moderate adaptation' (diagonal back-end of the surface), as projected by the 'Basic Linked System'. The projection on the left shows the effect of moderate adaptation to climate change (AL1), compared to the 'no adaptation' case.



**Figure 3:** Number of people at risk of hunger in Africa from 1990 until 2060 for a 'no climate change scenario' (front-end of the surface) and for a 'CO<sub>2</sub> doubling scenario with moderate adaptation' (diagonal back-end of the surface), as projected by the 'Basic Linked System'. The projection on the left shows the effect of moderate adaptation to climate change (AL1), compared to the 'no adaptation' case.

### Risk of Hunger in South and South-East Asia (AL1)



**Figure 4:** Number of people at risk of hunger in South and South-East Asia (not including China) from 1990 until 2060 for a 'no climate change scenario' (front-end of the surface) and for a 'CO<sub>2</sub> doubling scenario with moderate adaptation' (diagonal back-end of the surface), as projected by the 'Basic Linked System'. The projection on the left shows the effect of moderate adaptation to climate change (AL1), compared to the 'no adaptation' case.

Both approaches have their relative advantages. The risk of hunger indicator, as derived from the policy evaluation approach, is rather intuitive and its significance is easily understandable. The potential for adaptation can be investigated both at the farm level (e.g. by increased irrigation) and at a higher level (e.g. by changes in international food trade). The major disadvantage of this method, in the context of TWA, is the need to predefine a limited number of scenarios for climate change and socioeconomic development before the computationally demanding set of models can be run. Thus, we pay for the benefit of including the socioeconomic domain with the restriction to investigate only a limited number of scenarios. Moreover, it is not possible to derive the response functions directly from the outcomes of the analysis. E.g., if someone considers a 5% increase in the risk of hunger above reference level in any region at any time due to climatic change as intolerable, there is no straightforward method to translate this threshold into a regional tolerable climate window since the world regions are interconnected through the BLS.

In contrast, the sensitivity approach investigates biophysical impact variables (the so-called 'first order impact', like potential crop yield) without explicitly considering the connections to the socioeconomic system. The response surfaces shown in Figures 5 and 6 correspond directly to climate response functions. They translate an arbitrary regional climate state, represented by mean temperature, precipitation, and possibly other variables into the potential production for certain crops. Here, a tolerance limit in the impact domain corresponds directly to a tolerable climate window. E.g., if someone regards a 5% decrease in potential yield of a certain crop as intolerable, this can be directly translated into an isoline on the response surfaces (in the two-dimensional case). This would still not eliminate the need for assessing these biophysical response surfaces in the context of prevailing socioeconomic conditions in order to derive the region's climate vulnerability proper.

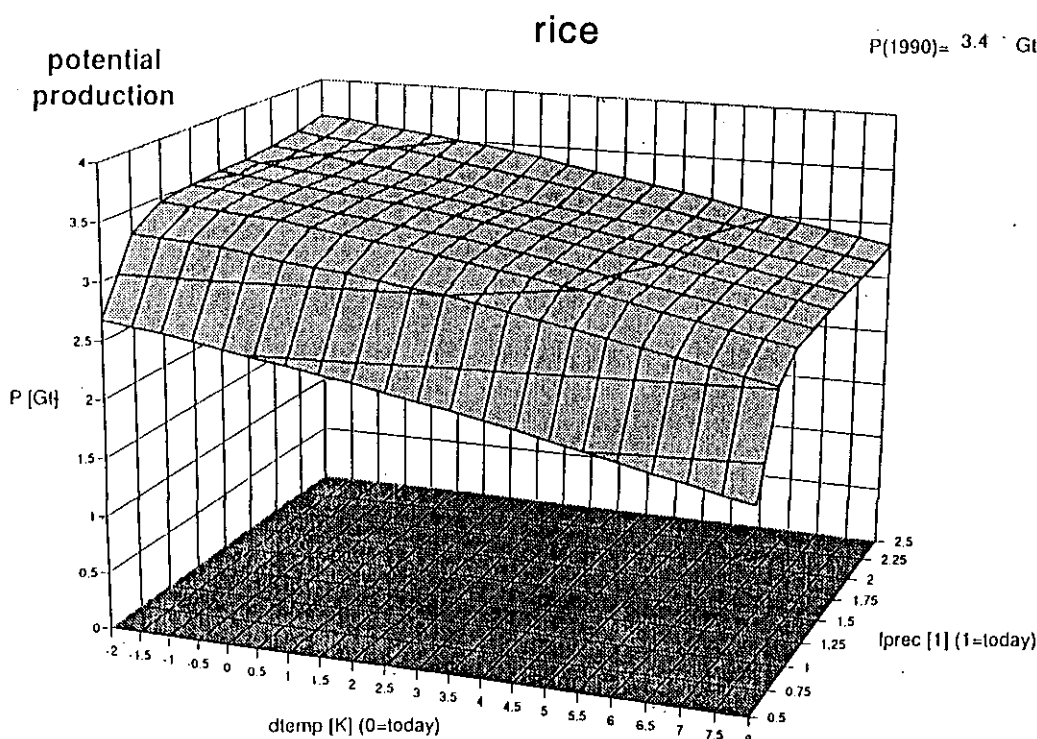


Both approaches have been widely used in climate impact assessments. In applying the TWA, it is important to separate direct biophysical effects on the one hand, and primary climate vulnerability and adaptive capacity on the other. This is relevant for most socioeconomic impact sectors, food supply and human health being the most prominent ones.

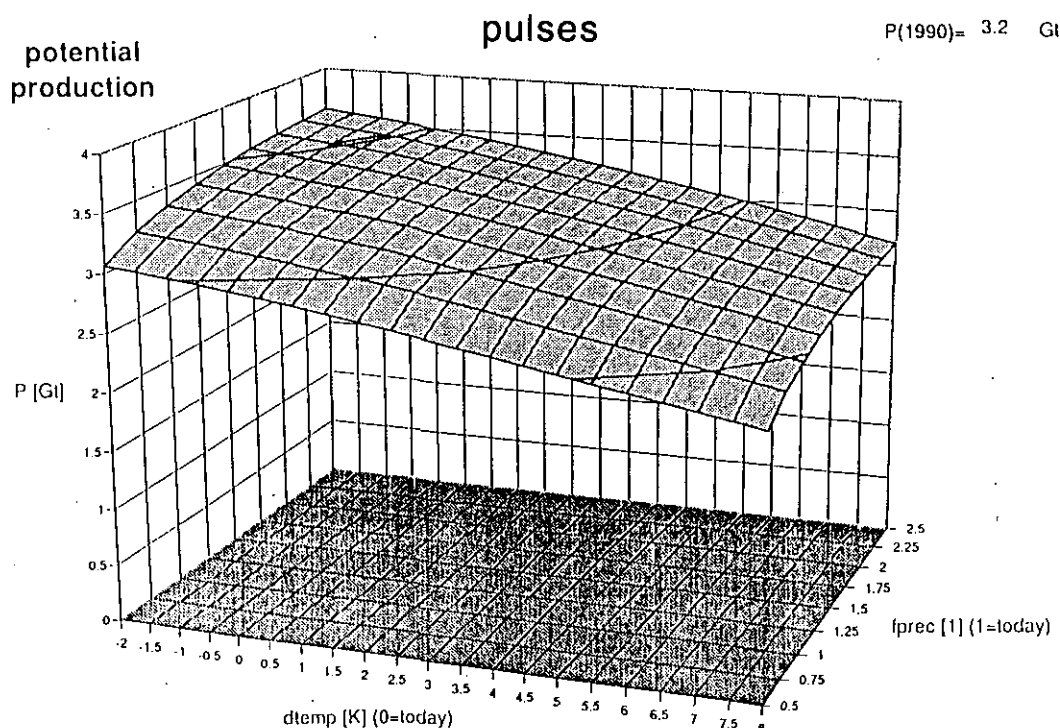
### Linking damages to climate windows

An important aspect of the TWA is that it should be possible to derive damage estimates associated with any specific TW. This exercise starts from a generalized heuristically defined climate window in terms of temperature (T) and rate of temperature change (DT). The particular window that will be analyzed later in the paper was formulated by WBGU (1995) (see also Petschel-Held et al., 1997) on the basis of historical temperature records and an expert assessment of potential climate impacts. This window is modified by additional constraints for sea-level rise (S) and its rate of change (DS) using actual figures according to Rijsberman and Swart (1990). The window is given by the constraints

$$\begin{aligned}
 &T \leq T_{\max} \\
 &DT \begin{cases} DT_{\max} & \text{if } T \leq T_{\text{trans}} \\ DT_{\max} \sqrt{\frac{T_{\max} - T}{T_{\max} - T_{\text{trans}}}} & \text{else} \end{cases} \\
 &S \leq S_{\max} \\
 &DS \leq DS_{\max}
 \end{aligned} \tag{3.1}$$



**Figure 5:** Potential crop yield (P) for rice in South and South-East Asia, for a range of climate states, described by changes in annual mean temperature (dtemp) and annual precipitation (fprec), as calculated by IMAGE 2.0.



**Figure 6: Potential crop yield (P) for pulses in South and South-East Asia, for a range of climate states, described by changes in annual mean temperature (dtemp) and annual precipitation (fprec), as calculated by IMAGE 2.0.**

with parameters  $DT_{\max}$ ,  $DS_{\max}$  for the maximal rate,  $T_{\max}$ ,  $S_{\max}$  for the absolute value, and  $T_{\text{trans}}$  for the transition value from linear to non-linear behavior of the underlying generalized response function. In this study, we will make five different assumptions concerning the tolerable climate window, with parameters given in Table 1.

**Table 1: Parameter values for the climate windows defined by Equation 3.1\***

Name	$DT_{\max}$ (°C/dec)	$T_{\max}$ (°C)	$T_{\text{trans}}$ (°C)	$S_{\max}$ (cm) rel. 1990	$DS_{\max}$ (cm/dec)
WBGU	0.2	16.6	15.6	--	--
WBGU SLR	0.2	16.6	15.6	30.0	3.0
LARGE	0.3	17.6	16.6	--	--
LARGE SL	0.3	17.6	16.6	30.0	--
LARGE SLR	0.3	17.6	16.6	30.0	3.0

\*The corresponding windows are shown in Figures 7a and 7b together with the contours of some first damage functions (Tol, 1995). Dashes indicate that the corresponding constraints are not effective.

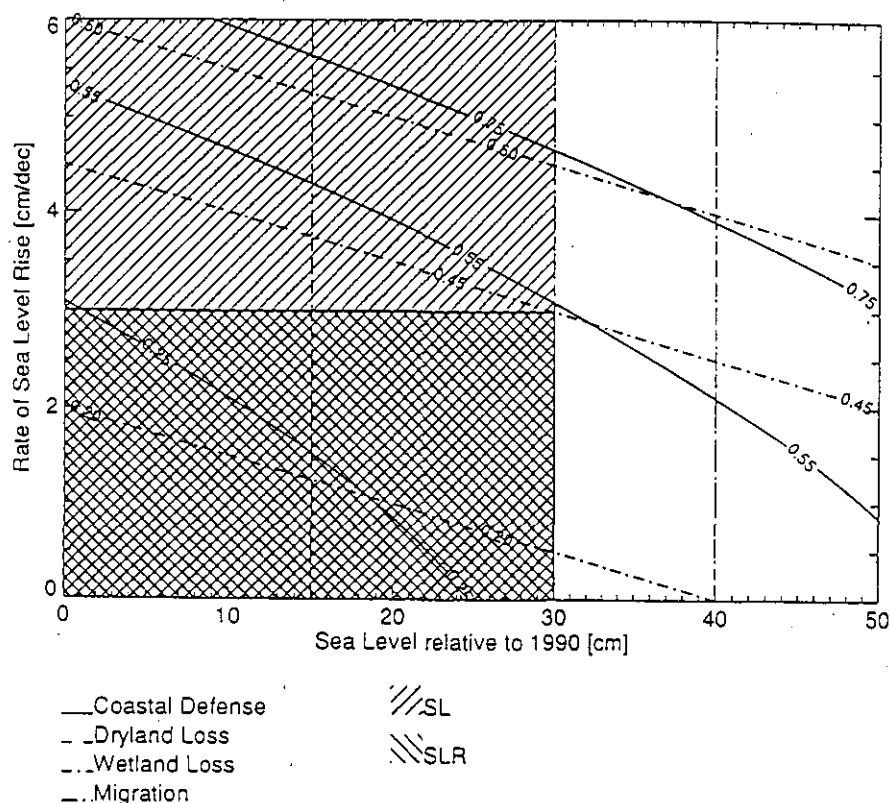
Note that this window specifies a quadratic relationship between tolerable temperature change and the rate of temperature change (in contrast to some other studies like, for example, Alcamo and Kreileman, 1996). Yet the domains used in their study can be obtained from Equations 3.1 by imposing  $T_{\text{trans}}=T_{\max}$ . This relationship reveals a reasonable approach to the non-linear relation between the damages due to the magnitude (T) and to the rate (dT/dt) of climate change. Parameters in Table 1 should be seen as scenario assumptions rather than well-founded thresholds.

In order to get some insight into the actual damages induced by climates assumed to be tolerable, we apply an earlier version of the damage module from the Climate FUND model (Tol, 1995). This damage module can be decomposed into two parts: a set of benchmark

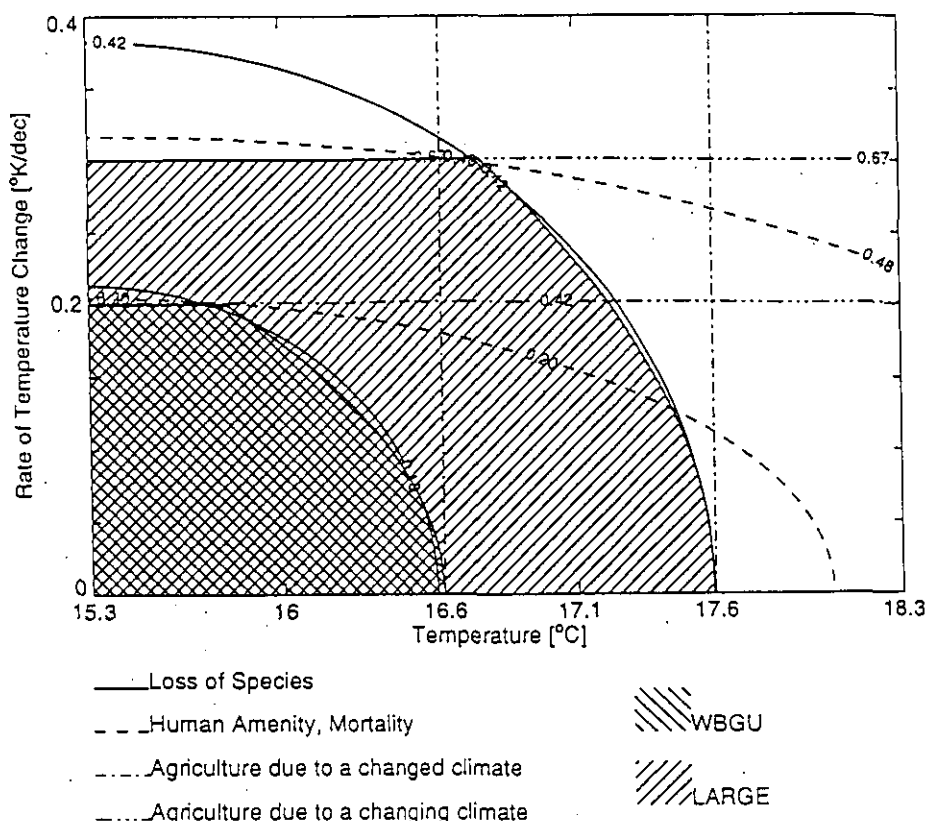
damages calculated for an effective CO<sub>2</sub>-doubling and an explicit formulation of damages as functions of the magnitude and rate of temperature change. Tol's model computes damages in *economic* units, i.e. US\$1990 or share of gross world product. In our case, however, we are more interested in the original *physical* units. Unfortunately, impacts and the economic value of a single physical impact unit are not delineated (except for Table 6.5 in Pearce et al., 1996, where damages for a CO<sub>2</sub>-doubling are listed in physical units). We therefore concentrate on these functions in terms of damages relative to benchmark values rather than their absolute values<sup>1</sup>. This is acceptable for the global analysis presented here as the functional behavior is taken to be homogeneous throughout the world.

Damages are assessed as quadratic functions of relevant climatic parameters (in this case magnitudes and rates of climate change and sea-level rise) through the origin. Damages due to the magnitude of climate change ( $X$ ) and due to the rate of climate change ( $dX/dt=DX$ ) have been divided by using expert estimates. The final forms (Tol, 1995) then allow are to plot the isolines of different damages into the (T,DT) and (S,DS)-plane, respectively. This is shown in Figures 7a and 7b where these isolines are overlaid with windows of tolerable climate evolutions specified in Table 1.

Note that agricultural damages are split into two components according to magnitudes and rates of climate change. This is due to the fact that for the latter adaptation is limited. Therefore it is not possible to describe agricultural damages with respect to one single benchmark value. Table 2 summarizes the damage thresholds for various impact categories corresponding to different climate windows in terms of relative damages.



**Figure 7a:** Isolines for four different damage categories depending on temperature change and its rate of change according to Tol (1995). Damages are indicated relative to the CO<sub>2</sub> doubling benchmark values which ideally are given in physical units (e.g. Pearce et al., 1996). Shaded areas depict the tolerable climate windows assumed in this paper. The windows are based on a review of historical temperature records and expert guesses.



**Figure 7b:** Isolines for damages depending on sea-level rise and its rate of change. Damages are indicated relative to the  $\text{CO}_2$  doubling benchmark (50cm absolute, 8.3cm/decade). Shaded areas depict the tolerable windows assumed in this paper. As our model underestimates sea-level rise to some extent, the assumed limit of 30cm is not too strong but rather lies in the middle of the interval [20cm,50cm] given by Rijsberman and Swart (1990). The functional forms for migration and dryland loss are identical, therefore the corresponding lines overlap.

**Table 2a:** Thresholds for climate impacts due to temperature change and its rate of change relative to the  $\text{CO}_2$ -doubling benchmark values.

Categories Names	Agriculture magnitude	Agriculture rate	Species Loss	Amenity, Mortality
WBGU	0.6	0.42	0.13	0.2
LARGE	0.92	0.67	0.42	0.48

**Table 2b:** Thresholds for climate impacts due to sea-level rise and its rate of change relative to the  $\text{CO}_2$ -doubling benchmark values.

Categories Names	Coastal defense	Wetland losses	Dryland losses	Migration
SL	0	0	0.6	0.6
SLR	0.55	0.45	0.6	0.6

It is clear that the damage analysis presented here is very crude and provides only weak indications for the damage levels assumed as thresholds. Nevertheless, these values might serve as useful illustrations for a heuristic definition of the climate window.

#### 4. Climate module: from tolerable climate change to admissible emission paths

At the current stage of the ICLIPS project, simple but powerful climate and greenhouse gas cycle models are used to describe the relationship between greenhouse gas emissions and climate change. We focus solely on carbon dioxide, the main contributor to anthropogenic climate change (IPCC 1996a). CO<sub>2</sub> serves as a proxy variable for all relevant greenhouse gas emissions in our calculations. Global temperature, rate of temperature change, global sea-level rise, and rate of sea-level rise are assumed to be adequate indicators to assess impacts of climate change. Later versions of the model will include other greenhouse gases (like CH<sub>4</sub> and N<sub>2</sub>O) and sulfur aerosols, as well as regionalized climate variables according to their role in regional vulnerability to climate change (IPCC, 1996b).

##### Carbon cycle model

The carbon cycle is described according to a simple model provided by another ICLIPS collaborator, Klaus Hasselmann (1995). The model represents the atmosphere-ocean CO<sub>2</sub> flux in a simplified manner. The amount of carbon dioxide in the atmosphere  $C(t)$  (measured in ppm) increases due to anthropogenic CO<sub>2</sub> emissions  $E(t)$  (in GtC/yr) produced by fossil fuel combustion, cement production, and land use change. CO<sub>2</sub> emissions are divided between the atmosphere and ocean as terrestrial biosphere uptake and geological processes are neglected in a first order approximation. We obtain

$$\dot{C}(t) = \beta E(t) - \beta \dot{D}(t), \quad (4.1)$$

where  $D(t)$  (in GtC) denotes the amount of carbon in the ocean. The conversion rate  $\beta$ , relating amounts of carbon to atmospheric concentration, is set equal to 0.47 ppm/GtC according to (Meier-Reimer and Hasselmann, 1987).

Carbon uptake by oceans, i.e., the net flux of CO<sub>2</sub> through the air-sea interface, is proportional to the difference between partial pressures of CO<sub>2</sub> in the atmosphere and in the ocean's mixed layer (Meier-Reimer and Hasselmann, 1987) which themselves are approximately proportional to the total additional amount of carbon  $C(t)$  and  $D(t)$ , respectively. Thus, we get

$$\dot{D}(t) = \rho[C(t) - C_0] - B[D(t) - D_0]. \quad (4.2)$$

$C_0 = 290$  ppm and  $D_0 = 37000$  GtC are the corresponding pre-industrial carbon levels and the parameters  $\rho$  and  $B$  are, in a first order approximation, assumed to be independent of temperature change.

Introducing the cumulative anthropogenic CO<sub>2</sub> emission  $F(t)$  (measured in atmospheric ppm), defined by

$$\dot{F}(t) = \beta E(t), \quad (4.3)$$

we obtain:

$$F(t) = [C(t) - C_0] + \beta[D(t) - D_0], \quad (4.4)$$

where, by neglecting the terrestrial biosphere and any geological processes, we have taken into account that all carbon released by human activities is either in the atmosphere or in the ocean.

Combining Equations (4.1), (4.2) and (4.4) yields:

$$\dot{C}(t) = \beta E(t) + BF(t) - \sigma[C(t) - C_0]. \quad (4.5)$$

$B = 3.2 \cdot 10^{-3} \text{ yr}^{-1}$  and  $\sigma = \beta\rho + B = 0.022 \text{ yr}^{-1}$  are chosen according to detailed carbon cycle calculations (Hasselmann, 1995).

In the following, Equations (4.3) and (4.5) are used as an efficient representation of the carbon cycle.

### Temperature evolution

The time evolution of the global annual mean surface air temperature  $T(t)$  (in °C) is calculated by (Budyko, 1974; Hasselmann, 1995):

$$\dot{T}(t) = \mu \ln \left[ \frac{C(t)}{C_0} \right] - \alpha(T(t) - T_0) \quad (4.6)$$

where the model parameters  $\mu = 0.087^\circ\text{C}/\text{yr}$  and  $\alpha = 0.017 \text{ yr}^{-1}$  are calibrated by using historical records of actual global temperatures and atmospheric  $\text{CO}_2$  concentrations and GCM results corresponding to a climate sensitivity of  $3.5^\circ\text{C}$ .  $T_0 = 14.6^\circ\text{C}$  denotes the preindustrial global mean temperature.

### Evolution of sea-level rise

Climate-related sea-level rise is caused mainly by thermal expansion of the ocean and increased melting of glaciers, ice caps, and ice sheets. Sea-level rise  $s(t)$  (in cm) relative to the pre-industrial level is therefore given by:

$$s(t) = s_{Th}(t) + s_{Gl}(t) + s_{Gr}(t) + s_{An}(t), \quad (4.7)$$

where  $s_{Th}(t)$  denotes sea-level rise due to thermal expansion,  $s_{Gl}(t)$  is sea-level rise caused by increased melting of glaciers and small ice caps, and  $s_{Gr}(t)$  and  $s_{An}(t)$  represent the contribution of the Greenland ice sheet and the Antarctic ice sheet, respectively.

Thermal expansion of the ocean is expected to be the main contributor to global sea-level rise over the next 100 years (IPCC, 1996a). It is described here by using a reduced-form model which is equivalent to a simple impulse-response model provided by Hasselmann et al. (1993):

$$\dot{s}_{Th}(t) = \alpha_s \ln \left[ \frac{C(T)}{C_0} \right] - \mu_s s_{Th}(t) \quad (4.8)$$

The corresponding impulse-response model parameters  $\alpha_s = 0.432 \text{ cm}/\text{yr}$  and  $\mu_s = 1/(99 \text{ yr})$  are calibrated to GCM results (ECHAM-1/LSG) and, therefore, take into account the full complexity of ocean-climate interactions as long as the linear regime is not abandoned. Equation (4.8) reflects the well-known time lag behavior of the thermal ocean expansion (IPCC, 1996a). The models we use to estimate contributions by glaciers and ice sheets are identical or similar to those applied by IPCC (1996a) (see Wigley and Raper, 1993; Warrick and Oerlemans, 1990).

### Climate model applications

The climate submodels described above can be applied in different ways. In the forward mode, we can calculate the temperature evolution caused by a certain emission scenario  $E(t)$  by solving simultaneously Equations (4.1) - (4.6), based on the following initial values for 1995:  $E(t=1995) = 7.9 \text{ GtC}/\text{yr}$ ,  $F(t=1995) = 200 \text{ ppm}$ ,  $C(t=1995) = 358 \text{ ppm}$ ,  $T(t=1995) = 15.3^\circ\text{C}$ . The calculated temperature and concentration changes can then be used to obtain the corresponding sea-level rise. Relevant initial values for sea-level rise in 1995 are determined by a simulation run starting in 1775 (pre-industrial era with vanishing contributions to sea-level rise) and by using time series of total radiative forcing (cf. Figure 6 in IPCC, 1996a). The initial values are close to the IPCC estimates and given by:  $s_{Th}(t=1995) = 3.8 \text{ cm}$ ,  $s_{Gl}(t=1995) = 5.14 \text{ cm}$ ,  $s_{Gr}(t=1995) = 1.9 \text{ cm}$  and  $s_{An}(t=1995) = 1.35 \text{ cm}$ .

An alternative way is to use these submodels in inverse mode, e.g., to start with a given temperature  $T(t)$  and deduce the corresponding emission path  $E(t)$ . One way of inverse calculation is to define a certain objective function which depends on temperature and/or sea-

level rise and to search for an emission path that maximizes (or minimizes) this objective function. This implies finding the cost-effective emission path related to a given temperature ceiling or to determine a cost-benefit solution where no additional constraints are taken into account. Another inverse calculation starts with setting a tolerable climate window, e.g., with respect to temperature change, rate of temperature change, sea-level rise, and rate of sea-level rise, and to deduce the set of all emission paths which are compatible with all constraints imposed by TWs.

In mathematical terms, the latter problem is an "ill-posed problem" (Tarantola, 1987) because, in general, the solution is not unique, i.e., there is more than one permitted emission path. But this "ill-posed" feature is a problem only when one is really interested in a single unique emission path. For TWA, it is more valuable to find the totality of all emission paths consistent with the TWs. This set of solutions consists of different emission paths, but the set itself is unique in a set theoretical sense. Deducing this set and finding appropriate ways to describe this solution is in the core of the TWA. Specific methods and first results are described in detail in Section 5.

### **The economic module**

The objective of the economic module in the TWA is to describe the relationships between economic activities, GHG emissions, and welfare implications of various reduction measures. Economic modules in integrated assessment models typically incorporate various control instruments in order to compare their economy-wide implications in achieving specific reduction targets.

While the ICLIPS team is developing its own economic model to perform these tasks, it was important to start experiments in integrating an already existing economic model into the framework of the TWA. This model should be able to identify a cost-effective emission path under the constraints provided by the predefined climate window. The MERGE model (Manne et al., 1995) was selected for these experiments.

MERGE is an integrated assessment model consisting of the economic model GLOBAL 2200, a simple climate model, and a damage assessment module. The economic model divides the world into five regions. It is an applied general equilibrium model. At each point in time, supply and demand are equilibrated through the prices of internationally traded commodities: oil, gas, coal, carbon emission rights, and a composite good representing the output of all non-energy sectors. MERGE is designed to run in an intertemporal optimization mode. It determines future emission paths by maximizing a welfare function. The model can be controlled by embedding constraints or targets for temperature change, greenhouse gas concentration levels and emission levels; or by defining exogenous climate policy instrument variables like carbon/energy taxes, regional shares in global carbon emissions and penalties for exceeding carbon limits. Potentials for emission reductions are modeled by considering the diffusion of new non-carbon energy technologies on the one hand, and by allowing substantial end-of-the-pipe emission abatement on the other.

## **5. Results from the first experiments**

In this section, the simple models described in the previous sections are applied to illustrate the possibilities and types of results produced by TWA. Besides two different representations for the complete set of emission paths admissible under the externally specified tolerable windows, we also present cost-effective emission profiles corresponding to the two global mean temperature windows. The latter type of result is well-known from other studies (Edmonds et al., 1995; Wigley et al., 1996), whereas the representation of the entire set of

admissible emission profiles is propagated only more recently (Alcamo and Kreileman, 1996; Swart et al., 1996; Petschel-Held et al., 1997). Methods to obtain and describe what in mathematical terms is called the *funnel of admissible solutions* are described in more detail elsewhere (Bruckner and Petschel-Held, 1997). Here, we just give a short sketch of the mathematics and put more emphasis on the actual results obtained within the simple model framework outlined in Sections 3 and 4.

### Admissible emission paths: theory and methods

Suppose there is an ideal model for the climate system in terms of damages, climate and atmospheric variables, and socioeconomic mitigation costs represented by the state vector  $\mathbf{x}$ . The time evolution of the state vector depends on a set  $\mathbf{u}$  of instruments and measures (e.g. taxes, tradable permits, education programs, etc.), i.e.,

$$\dot{\mathbf{x}} = \frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \quad (5.1)$$

In this simple model, the state vector  $\mathbf{x}$  is given by the annual emissions (E), cumulative CO<sub>2</sub> emissions (F), the CO<sub>2</sub> concentration in the atmosphere (C), the global mean temperature (T), and four variables for different contributions to sea-level rise (S)<sup>2</sup>. TWs for the different variables are expressed as a restriction on the values of the state vector, i.e.  $\mathbf{x} \in \mathbf{D}$  where  $\mathbf{D}$  is the intersection of all tolerable windows. In our case, the domain  $\mathbf{D}$  is given by Eq. (3.1). There is, however, a further constraint for the control variable  $\mathbf{u}$  which is given by the reduction rate of the CO<sub>2</sub> emissions. This constraint should be obtained from socioeconomic analysis, but here we apply simple assumptions. In general, the constraint can be written as  $\mathbf{u} \in \mathbf{U}(\mathbf{x}, t)$  where  $\mathbf{U}(\mathbf{x}, t)$  represents the TW for the control variable. By inserting the constraint into Eq. (5.1) we get the basic equation for TWA

$$\dot{\mathbf{x}} \in \mathbf{F}(\mathbf{x}) = \{\mathbf{f}(\mathbf{x}, \mathbf{u}) | \mathbf{u} \in \mathbf{U}(\mathbf{x}, t) \wedge \mathbf{x} \in \mathbf{D}\}. \quad (5.2)$$

Here  $\mathbf{F}(\mathbf{x})$  is a set valued function, often simply called a *multi*. Recently, equations like Eq.(5.2) have attracted attention in control theory where they are called *differential inclusions* or multivalued differential equations (Aubin and Cellina, 1984; Deimling, 1992).

A number of mathematical features of differential inclusions are important in our case as they are concerned with the existence and properties of its solutions. Skipping the mathematical details, we note that the problem has a solution with a compact and connected solution set if sets  $\mathbf{F}$  and  $\mathbf{D}$  are closed convex and some further, rather weak, conditions are fulfilled. The tube of all admissible solutions is called the *funnel*. A discrete representation of the boundary of the funnel, i.e. build up from a finite yet sufficient number of points, can be computed by a rather simple, yet nontrivial, algorithm (for details see Bruckner and Petschel-Held, 1997):

1. Start from the initial condition  $\mathbf{x}_0$ , choose a time step  $\Delta t$  and set  $I=0$  where  $I$  is the number of 'switches' between different controls. Specify a control rate  $\hat{\mu}_1$  where  $\hat{\mu}_1$  is an extreme value of  $\mathbf{U}(\mathbf{x}, t)$ . Compute the number  $N$  of possible switches between extreme control rates analytically, if possible. If not, start with a low  $N$  (in general  $N=2$  is a good starting point), compute the funnel and repeat the procedure for  $N=N+1$  until the result converges.
2. Integrate Eq.(5.1) one time step from  $t$  to  $t+\Delta t$  with the current control rate  $\mu_1$ . If  $I < N$  do the same with controls  $\mu$  as the other extremes of the control set  $\mathbf{U}(\mathbf{x}, t)$ <sup>3</sup> and set  $I=I+1$ .
3. Check, whether the condition  $\mathbf{x} \in \mathbf{D}$  is fulfilled; if not skip the orbit from the list of boundary orbits.
4. Check, whether the continuation of the actual state vector with  $\mathbf{u}=\mathbf{u}_{\min}$  stays within  $\mathbf{D}$  for times large enough to reach a quasi-equilibrium state.
5. If  $t+\Delta t$  is large enough stop; else set  $t=t+\Delta t$  and go to 2.



In this manner a rather large, yet treatable, number of boundary orbits is computed which allows to extract the intended information on the funnel.

### Admissible emission paths: necessary conditions

We have applied the algorithm described above to obtain the funnels for the windows described in Section 3. The following constraints characterize the emission functions:

1. smoothly differentiable, i.e.  $E(t) \in C$ , with a limited value for the derivative chosen to be  $|\dot{E}|/E \leq 10\% / \text{yr}$ ;
2. monotonously decreasing after switching to effective reduction measures, i.e. having chosen a decarbonization strategy there is no possibility to increase emissions again; and
3. the initial growth is limited by the current growth rate of about 2% per year.

Note again, that the constraints for the emission profiles have actually to be expressed in more 'natural' units like welfare losses, and therefore should be embedded into a socioeconomic model. The constraints chosen here, though reasonable, are heuristic.

In order to visualize the results, we have projected the funnel onto the five main state variables: emissions  $E$ , cumulative emissions  $F$ ,  $\text{CO}_2$  concentrations  $C$ , global mean temperature  $T$ , and sea level  $S$ . Figures 8 and 9 show the resulting projections for the two temperature constraints WBGU and LARGE, respectively. Restrictions for sea-level rise are encoded by the solid (no restrictions), dotted (restriction for the level only), and dash-dotted lines (restrictions for both sea-level rise and its rate of change) for the upper boundary of the funnel. The depicted corridors represent necessary conditions for the time evolutions of the respective variables to fulfill the specified socioeconomic and climatic constraints: each admissible evolution lies within the corridor, but not any arbitrary function lying within the limits is admissible. On the other hand, any *point* within the domain can be reached by at least one admissible control function  $E(t)$ .

The main results for the WBGU domain are:

- Effective emission reduction has to commence in 2020 at the latest due to the limitation on the rate of temperature change. The reduction has to be rather sharp (see Figure 8).
- Sea-level rise is only relevant beyond the year 2100 due to restricting its absolute level to 30 cm above the 1990 value. The chosen rate of sea-level rise does not influence the results. Note that our model underestimates sea-level rise compared to the IPCC projections (Hasselmann et al., 1993). Therefore, the restriction to 30 cm seems to be not too strong.
- Without constraints for sea level, admissible concentration achieves a maximum of about 465 ppm for the period 2050 to 2100. It decreases to an equilibrium concentration of about 440 ppm afterwards.
- The admissible equilibrium value<sup>4</sup> for  $\text{CO}_2$  concentration in the case of a restricted sea-level rise is lowered to about 400ppm. In this case, cumulative emissions beyond 1995 are limited to about 640 GtC. Admissible temperature reaches a maximum of about 16.5 degrees celsius and then decreases to approximately 16 degrees celsius.

The LARGE window of tolerable temperature evolutions implies the following main differences in the results:

- The latest year for an effective emission reduction changes to 2042 without constraint on the sea level; 2039 with restriction on the absolute value of sea-level rise; and 2022 with restrictions on the rate of sea-level rise. In the latter case, however, the reduction is much less radical than for the WBGU domain. Yet, the restriction is effective until 2060 beyond which the corresponding emission limit is again determined by the absolute value of sea-level rise.

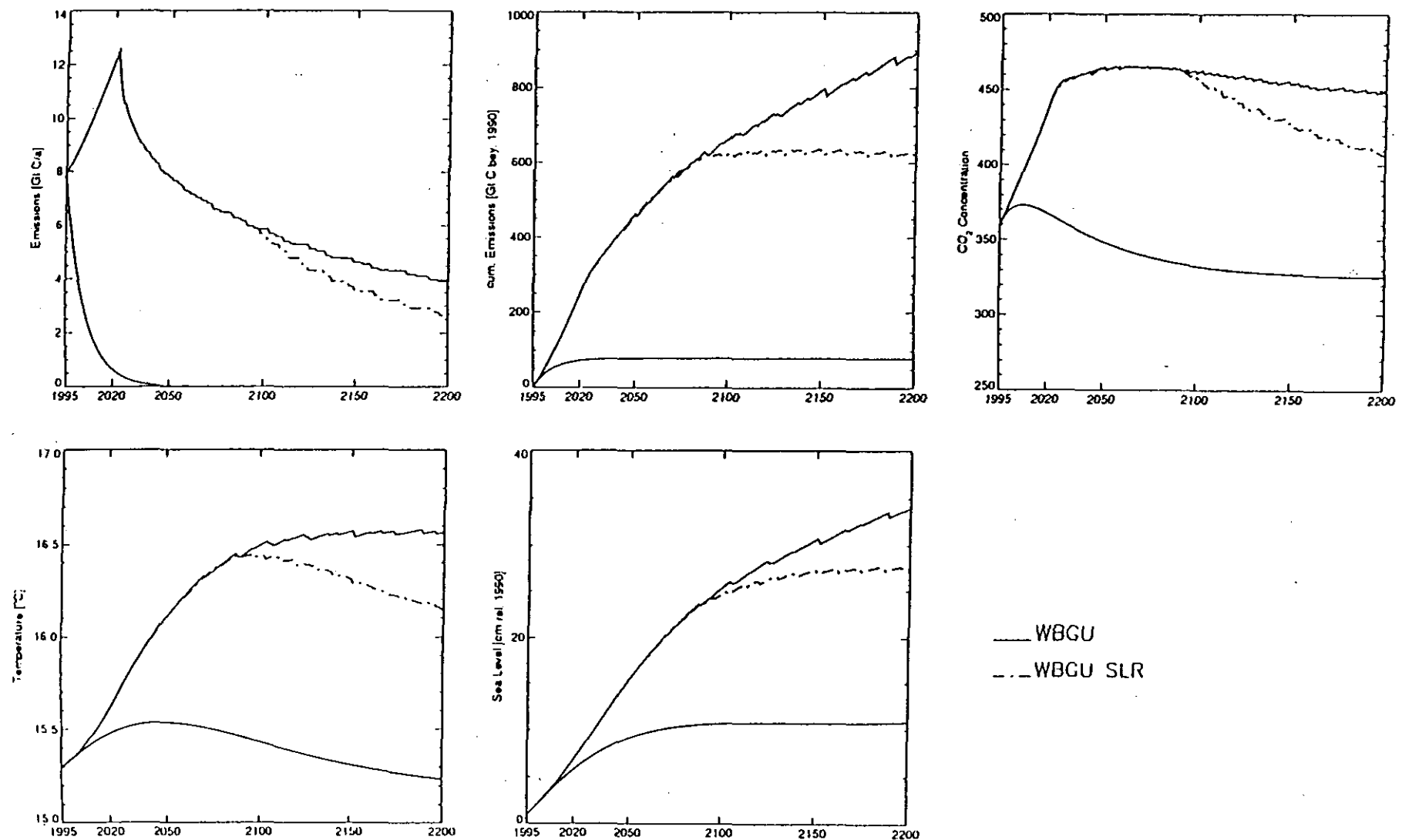


Figure 8: Projections of the funnel of admissible solutions for the WBGU temperature window ( $T_{max}=16.6^{\circ}\text{C}$ ,  $DT_{max}=0.2^{\circ}\text{C/decade}$ ) onto emissions, cumulative emissions, concentration, temperature, and sea level. The 'corridor' are delimited by the thick lines, where the line style characterizes the assumptions on tolerable sea-level rises.

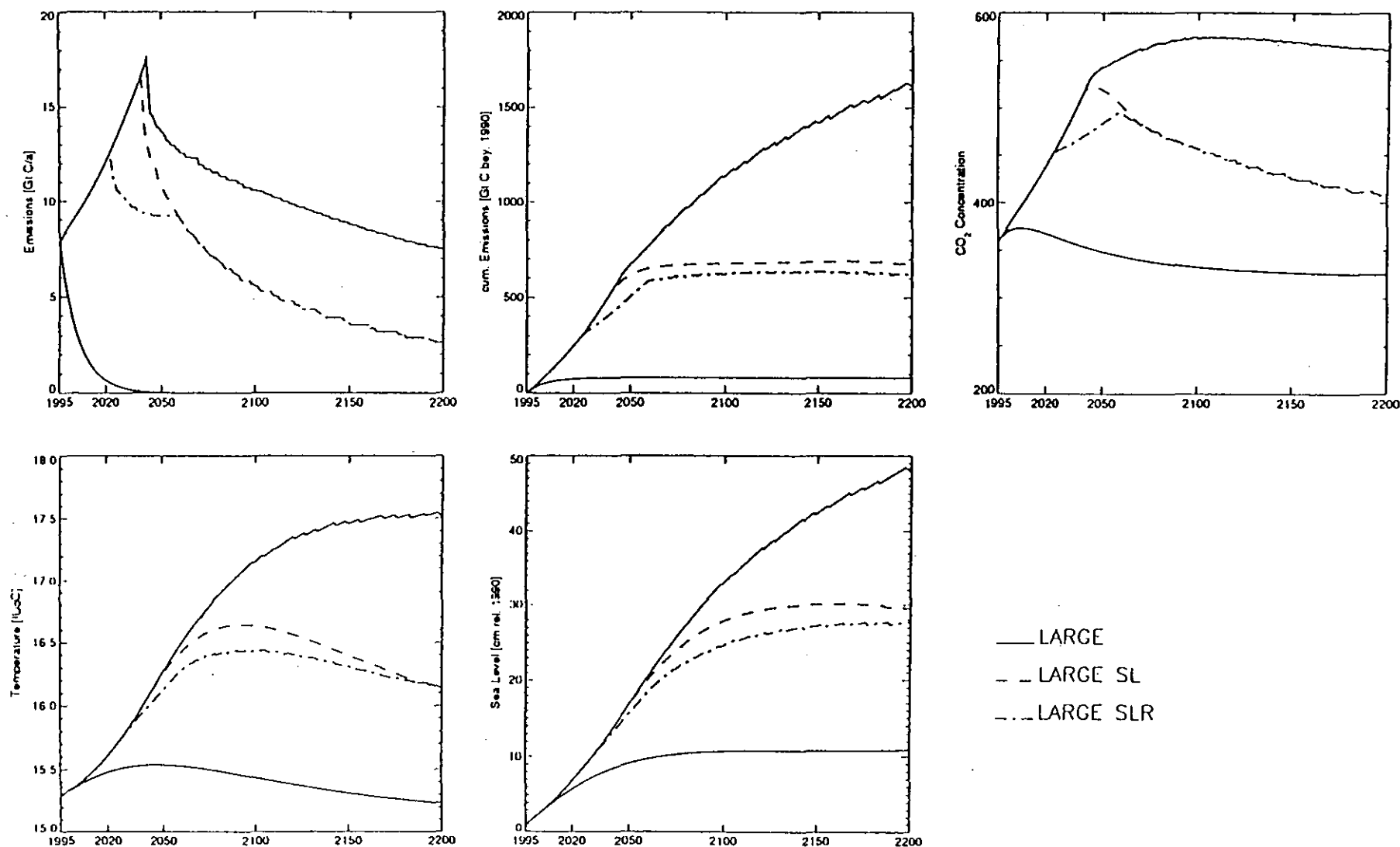


Figure 9: Projections of the funnel of admissible solutions for the LARGE temperature window ( $T_{\max}=17.6^{\circ}\text{C}$ ,  $DT_{\max}=0.3^{\circ}\text{C/decade}$ ) onto emissions, cumulative emissions, concentration, temperature, and sea level. The 'corridor' are delimited by the thick lines, where the line style characterizes the assumptions on tolerable sea-level rises.

- Without a confinement on the sea level, the maximal value of admissible CO<sub>2</sub> concentrations goes up 575 ppm, with a final equilibrium value of about 520 ppm (not shown).

Table 3 summarizes our main results in terms of the maximum admissible CO<sub>2</sub> emissions for the different assumption on tolerable windows. As one can see, emission reduction before 2030 is needed only if either the rate of temperature change or the rate of sea-level rise have to be restricted. Limitations on the absolute values are important only in the long term.

Table 3: CO<sub>2</sub> emission paths for different climate windows

CO <sub>2</sub> emissions in GtC/yr	2030	2050	2100	2200
WBGU	9.7	7.8	5.9	3.9
WBGU SL	9.7	7.8	5.6	2.6
LARGE	14.3	13.7	10.6	7.5
LARGE SL	14.3	10.7	5.6	2.6
LARGE SLR	10.2	9.2	5.6	2.6

### Admissible emission paths: sufficient conditions

Funnels represent necessary conditions which must be fulfilled by admissible policy paths. Hence, funnels are useful tools for filtering out those evolutions of investigated variables which are inconsistent with restrictions imposed by the normative setting of tolerable windows. But funnels are by no means "safe" corridors as has been discussed above. It is therefore necessary to give a sufficient description of the totality of all admissible policy paths which allows investigating the internal structure of funnels. One possibility is to use an appropriate parameterization of possible control functions, e.g. emission paths. This parameterization may be based on special function classes like orthogonal polynomials or pade approximants. The selected parameterization must fulfill several requirements. In order to be illustrative, it should involve only a few, e.g. three, parameters which should be as intuitive as possible. The parameterization should incorporate the insights gained from the necessary conditions, i.e., it must reflect the shape of the investigated funnels. Furthermore, it should avoid implausible oscillations or discontinuities and it should fulfill political confinements of climate control, e.g., smooth transitions to an effective emission reduction. These criteria can not solely be based on scientific grounds. They involve explicit normative settings like the definition of the tolerable window boundaries.

Results from the first experiments with the WBGU climate window are based on parameterizing annual CO<sub>2</sub> emissions (GtC/yr) as follows:

$$E(t) = \begin{cases} E_0(1 + a_0 t), & 0 \leq t \leq t_1 \\ e_1 + e_2 t + e_3 t^2, & t_1 \leq t \leq t_2 \\ e_4 \exp(-\gamma t), & t_2 \leq t \end{cases} \quad 5.1$$

where  $t_1$ ,  $t_2$  and  $\gamma$  are the basic parameters determining the form of  $E(t)$ .

Up to time  $t_1$ , the emission profile is assumed to follow the business-as-usual path, which can be characterized by a simple linear projection with an initial value of  $E_0 = 7.9$  GtC/yr in 1995 and an initial growth rate of 2%/yr (see Grubb et al., 1995). Following a transition period of  $(t_2 - t_1)$  years, the economy shifts to an exponential emission decrease with a reduction rate of  $\gamma$ .

$\Phi(t) = e_1 + e_2 t + e_3 t^2$  describes a parabolic spline interpolation between  $t_1$  and  $t_2$ . The remaining parameters  $e_1$ ,  $e_2$ ,  $e_3$  and  $e_4$  are fixed for a given set of  $t_1$ ,  $t_2$  and  $\gamma$  according to the smoothness

requirements of the spline interpolation. Figure 10 shows some typical emission profiles for different sets of  $(t_1, t_2, \gamma)$ . By scanning the entire reasonable part of the corresponding parameter space and putting the resulting functions into the differential equations of our model, the time evolution of all impact relevant variables (like temperature change, rate of temperature change, sea-level rise, etc.) can be computed and compared to the constraints imposed by the fundamental TWs. Adopting the theory of differential inclusions, it can be shown that this discrete scan of parameters may be generalized to the actually continuous parameter space (Petschel-Held et al., 1997). Using this procedure, we obtain a regime in the parameter space which encodes all admissible emission profiles corresponding to a maximum value  $\gamma_{\max}$  of the emission reduction rate.

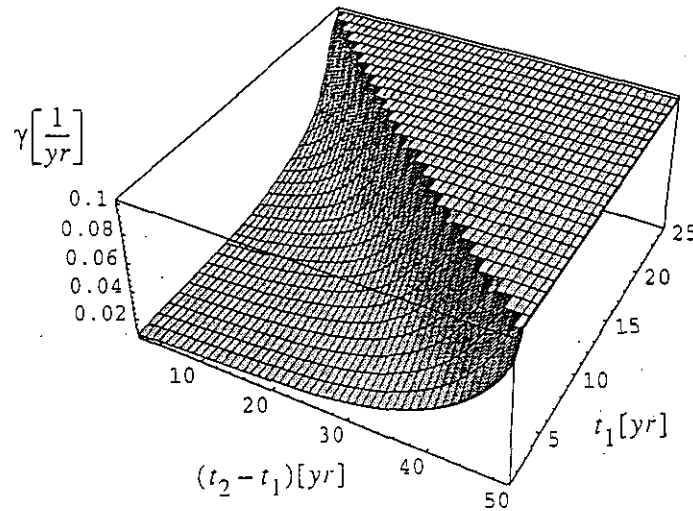


Figure 10: Examples of limit emission profiles for the WBGU domain.

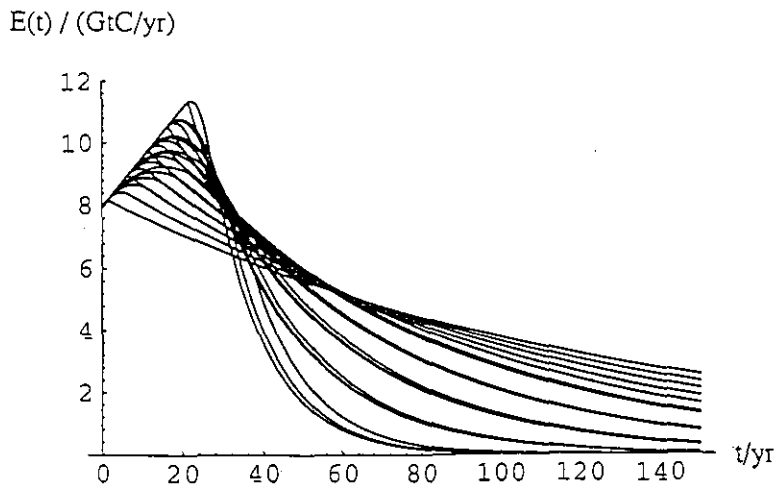


Figure 11: Surface of parameter values  $\gamma_{crit}$  separating admissible emission profiles (above) from forbidden paths (below) for the WBGU domain.

Figure 11 shows  $\gamma_{crit}$  as a function of  $t_1$  and  $(t_2 - t_1)$ , where  $\gamma_{max} \geq \gamma \geq \gamma_{crit}$  denotes admissible controls and where we have chosen  $\gamma_{max} = 10\%/yr$  as a scenario parameter for illustrative purposes only. According to Figure 11 the following results are obtained from our simple TW exercise:

- For a given largest acceptable value of  $\gamma$ , the time  $t_1$  before actually leaving business-as-usual and the length  $t_2 - t_1$  of the transition period are both restricted. The time to start a continuous reduction path of at most 10% per year is absolutely limited to 25 years. After that period, an instantaneous shift to an exponential decrease of emissions, by a constant rate of 10%/yr without any transition time, would be unavoidable. If we left the business-as-usual path right now, we would have a 50 year time span for a smooth transition to the reduction path. Although a reduction rate of 10%/yr might be theoretically possible, policymakers might prefer a lower maximum emission reduction rate, e.g. a rate of 2%/yr, taking into account additional social issues. According to the numerical results underlying Figure 11, a maximum emission rate of 2%/yr would correspond to a maximum allowable time span of only 15 years along the business-as-usual path (and no additional time for the transition to exponential decrease). Alternatively, we could leave the business-as-usual path right now and this yields a maximum length of about 30 years for the transition period.
- There is a clear trade-off between the time we follow the business-as-usual path and the length of the transition period to an exponential emission decrease. If we reduce the time  $t_1$  by 1 year we earn 2 additional years for the transition period. Since the sum of  $t_1$  and  $t_2$  is definitely restricted to about 50 years, this suggests that some might prefer keeping the time  $t_1$  shorter, i.e., leave business-as-usual earlier rather than later.
- It is necessary to reduce emissions by at least 0.7% per year, even if we start reduction now and if we would like to keep the reduction rate as small as possible now and in the long run.

In order to get an insight in the internal structure of the emission funnel, it is illustrative (see Figure 10) to plot all emission profiles which belong to parameter sets lying on the "surface" of Figure 11. These profiles are therefore characterized by the minimum emission reduction rate which is compatible with a given time  $t_1$  and a given length of the transition period  $(t_2 - t_1)$ . Emission profiles in Figure 10 are qualitatively very similar to the derived emission paths for stabilizing greenhouse gas concentration (see, e.g., Richels and Edmonds, 1995). Quantitatively, the constraints imposed by the chosen TW and the restrictions of the chosen class of emission paths lead to a reduction which must start in about 25 years at the latest, in contrast to Richels and Edmonds (1995).

We have conducted a simple sensitivity analysis with the climate model. Figure 12 shows  $\gamma_{crit}$  as a function of  $t_1$  for a zero length transition period. Since a reduction of more than 100%/yr seems to be really unrealistic, the time we follow business-as-usual is absolutely restricted to 27 years. Therefore, if we do not want to violate the chosen tolerable climate window and if we assume that the given class of the emission path is an appropriate one, we have to start with emission reduction in 27 years at the latest and reduce emission to nearly zero in the long run, whatever the economic potential of our societies may be. (Comment: the reduction to nearly zero follows from the climate model and not only from the specified exponential decrease.)

### Economic implications

According to the idea of the TWA, the first experiments aimed at the transformation of selected climate windows into tolerable emission corridors and emission paths. The MERGE model is used to incorporate requirements and restrictions of the economic system in these first experiments. Emission corridors result from superimposing the climate window with an economic constraint which represents the admissible economic efforts and expenditures. Going beyond the pure emission reduction rate (as relevant economic constraint in the previous subsection) these expenditures are measured as welfare losses related to an hypothetical reference path without climate change. Within MERGE, the economic system (especially the

energy subsystem) is modeled as a highly flexible and adaptive one. Hence, the emission corridor spreads immediately. Given the WBGU climate window as well as a maximum of 1% admissible welfare loss, the upper edge amounts to 9.6 GtC carbon dioxide emissions (excluding emissions from land-use change) in year 2000, while the lower edge amounts to 4.1 GtC (the initial value is 6.1 GtC in 1990).

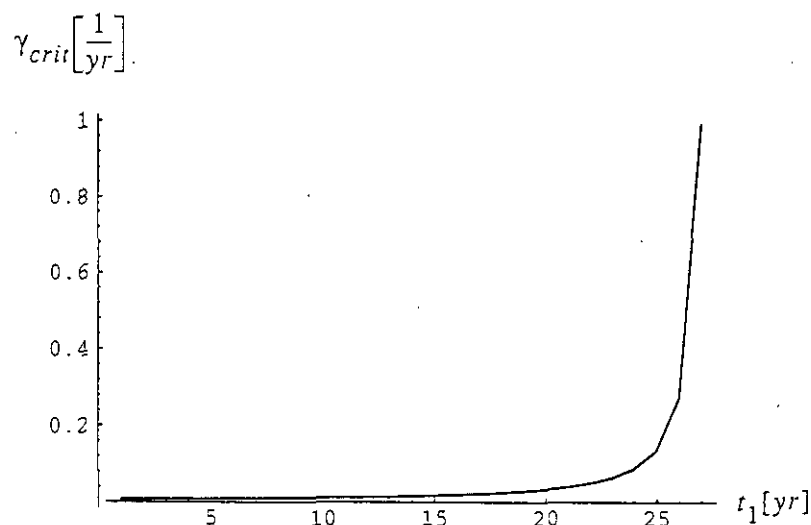


Figure 12:  $\gamma_{crit}$  as a function of  $t_1$  for a zero length transition period for the WBGU window.

This range of tolerable emissions is partly due to the enormous potential of end-of-the-pipe carbon dioxide abatement technologies assumed in MERGE.

While little suited for the determination of the whole emission corridor (due to its large computational time requirements), MERGE will be useful in determining specific future emission trajectories. We have applied MERGE in combination with two different climate windows (WBGU and LARGE) in order to calculate emission trajectories leading to a welfare optimum.

Emission trajectories based on these windows are demonstrated in Figure 13 together with two emission paths, determined by Manne and Richels (1995). The latter represent welfare optimizing emission trajectories which will stabilize atmospheric  $\text{CO}_2$  concentrations at the level of 450 and 550 ppm, respectively. Once started, the rapid decline of emissions is typical for all model runs. The WBGU climate window requires reducing  $\text{CO}_2$  emissions to almost 0 GtC by 2050. The emission path associated with the LARGE window follows the reference path until 2020 and declines from about 12 GtC to nearly 0 GtC in about 70 years.

The shape of these trajectories is similar to those derived from other climate policy models. Such shape implies to draw benefit from emission increases as long as possible, while simultaneously improving the economic base (capital stock), and then reduce emissions as fast as technological progress and capital turnover permit. Selected results characterizing the above emission paths are summarized in Table 4.

Table 4: Model results associated with various climate windows and concentration targets

	<i>Reference</i>	<i>WBGU Climate Window</i>	<i>LARGE Climate Window</i>	<i>Stabilization 450 ppm</i>	<i>Stabilization 550 ppm</i>
Welfare <sup>a</sup> (as % of reference value)	2466.7	2450.6 (99.35)	2463.8 (99.88)	2461.4 (99.78)	2464.6 (99.9)
$\Delta T$ in 2200 <sup>b</sup>	5.56	1.3	2.3	2.05	2.8
CO <sub>2</sub> -concentration in 2200 (in ppm)	1285	358.5	483	450	545

<sup>a</sup>in arbitrary units

<sup>b</sup>The 1990 value represents the reference base of temperature change; it exceeds the preindustrial level by 0.7 C.

Despite the higher cumulative emission in the 550 ppm stabilization scenario compared to the LARGE Window scenario (Figure 13), the welfare gain is only marginal (2464.6 versus 2463.8). Since this surplus of emissions appears mainly after 2100, the reason of this minor difference must be the discounting effect. The greatest difference is between the WBGU climate window and all other scenarios. However, the difference is still less than 1%, which might be somewhat unexpected considering the early and drastic emission reductions. Even the difference relative to the welfare value (2466.7) of the reference path (not shown in Figure 13) is just marginal. This is surprising insofar as the reference emission path is increasing up to 30 GtC in 2120. This again documents the considerable adaptability of the economic system assumed in MERGE. Some climate parameters, however, significantly differ across the four scenarios (see temperature increase and CO<sub>2</sub> concentration in Table 4).

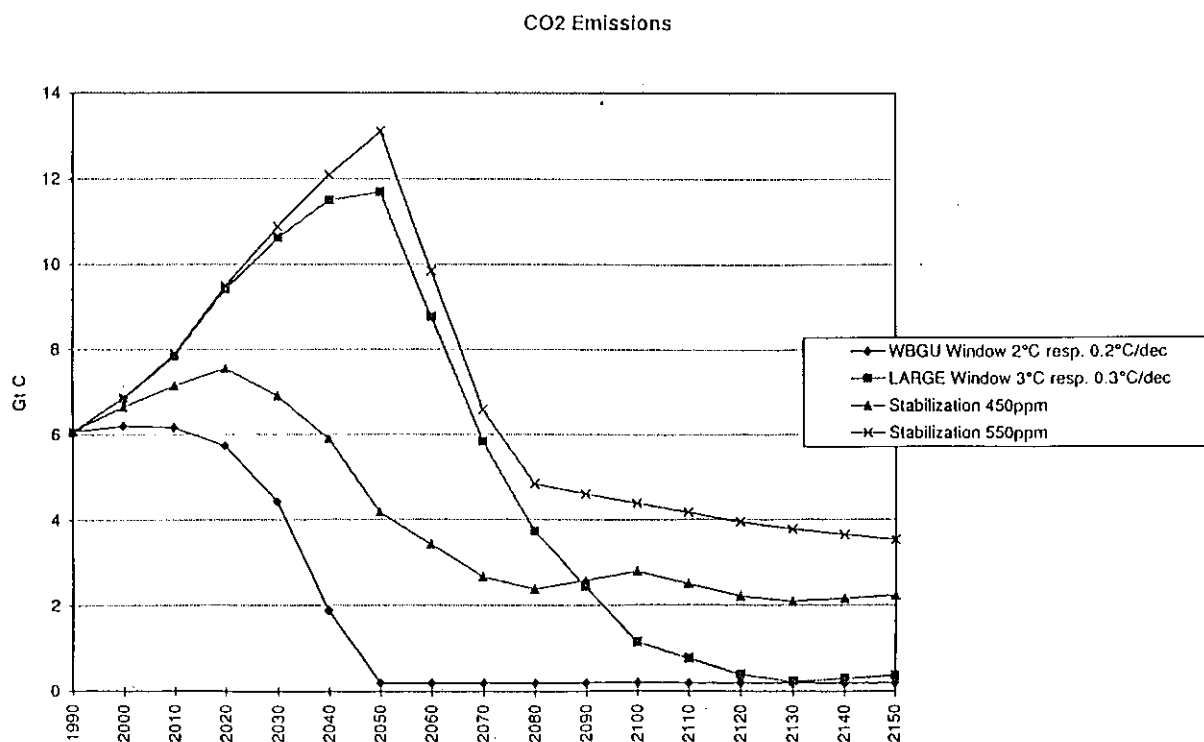
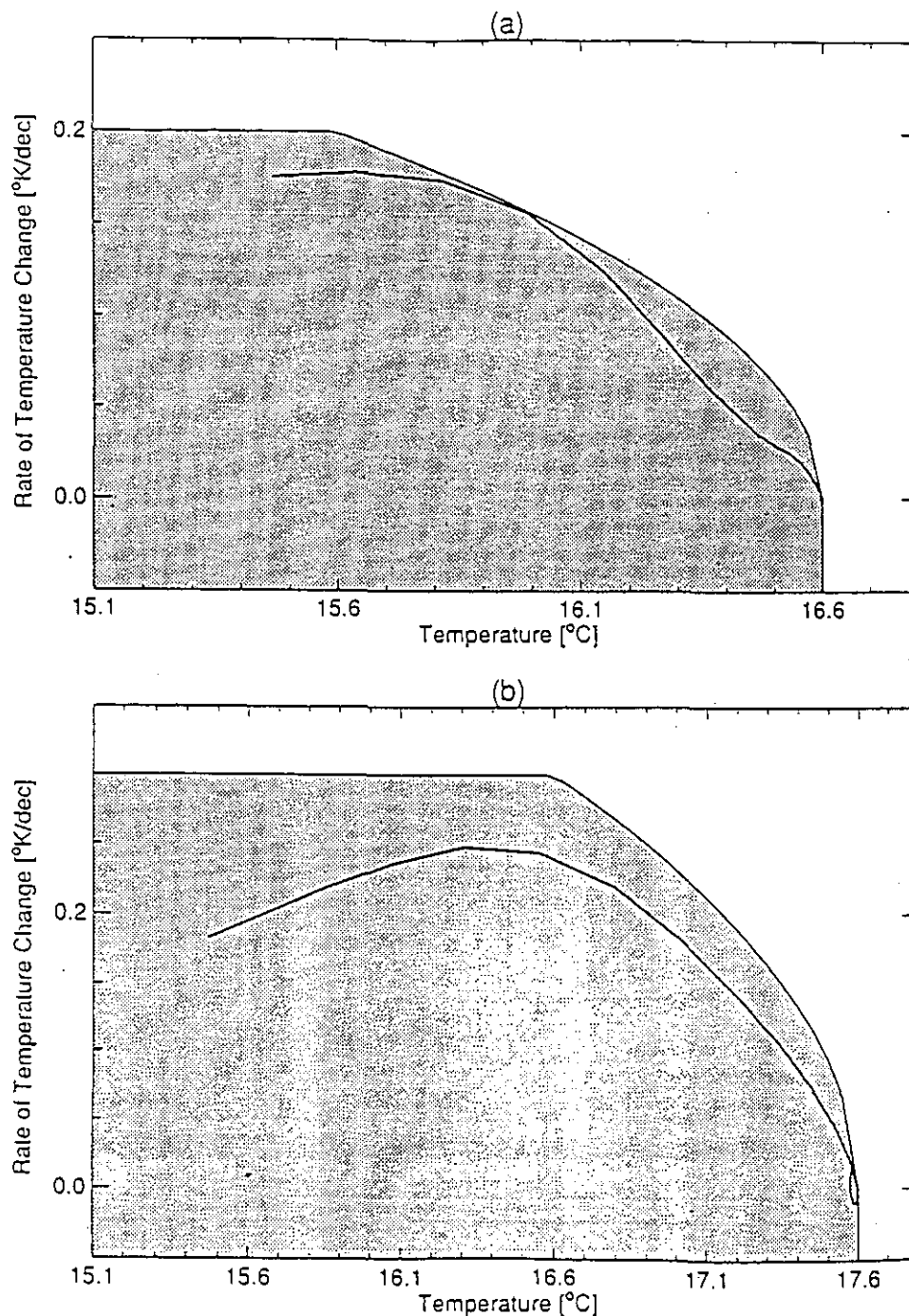


Figure 13: Welfare optimal emission paths.





**Figure 14: Trajectories of the welfare optimal paths for the WBGU (a) and the LARGE (b) domains.**

The phase diagrams (Figure 14) demonstrate how the tolerable window scenarios move through the climate window. There is an almost direct way to the boundaries of the windows. While the boundary of the WBGU window will be touched very soon, the LARGE window will be approached gradually until the lower right corner is reached (this corner represents minimum temperature change rate and maximum absolute temperature change).

## 6. Summary and next steps

The ICLIPS project at PIK seeks to provide new insights in the problem of global climate change by pursuing the TWA. It is based on external normative specifications of tolerable sets of climate impacts as well as proposed quotas and policy instruments for implementation. The related “tolerable windows” are derived successively in a “backwards mode”. Our initial objective was to test the various concepts, approaches and modeling tools in these first experiments. As we have emphasized throughout the paper, our results are preliminary but interesting.

The objective of the TWA is twofold: first, to determine complete sets of all admissible emission paths which are compatible with the normative inputs and, second, to select optimal emission paths in a “second best” manner. While the second task can be embedded in the usual cost-effectiveness framework, the first one requires an appropriate mathematical foundation which is based on the theory of differential inclusions. The methodological tools we developed appropriately take into account the dynamic aspects of the entire problem, e.g. restrictions on the rate of climate change or socioeconomic conditions. From this point of view, the TWA may be considered as a dynamic generalization of the critical load concept. The solution methods are suitable for providing necessary and sufficient descriptions for the complete set of all admissible emission paths, i.e. for all emission paths which are compatible with the predefined windows. The boundaries of the tube of all admissible paths (called a “funnel” in mathematical terms) are calculated for different climate windows and socioeconomic restrictions. The funnels are by no means “safe” corridors. Although all admissible paths lie within the corridor (=funnel), not every arbitrary path lying within the limits of the corridor is admissible. The required insight into the internal structure of the funnels is gained by implementing an appropriate parameterization of the emission profiles. Finally, the selection of cost-effective emission paths is conducted with the MERGE model in accordance with our climate windows.

The main results gained by our first TWs exercise can be summarized as follows. In order not to violate the restrictions imposed by a maximum temperature change of 2°C and a maximum rate of temperature change of 0.2°C per decade, and in order to allow a smooth transition to a fossil fuel free economy, we have to reduce greenhouse gas emissions in 25 years at the latest, if an emission reduction rate of 10% per year should not be exceeded. However, in this case the reduction in 2020 has to be rather sharp. The time we can follow the business-as-usual path is absolutely restricted to 27 years. If we want to avoid a reduction rate larger than 2% per year, we have to start abatement 15 years from now at the latest. For each year we continue along the business-as-usual path, we lose 2 years for a smooth transition to an exponential emission decrease. In any case, GHG emissions must reach nearly zero in the long run. In the cost-effective case, a level of nearly zero is already reached in the middle of the next century. The welfare loss of this optimal path is smaller than 1% of the respective reference value. In the short run, i.e. up to about 2030, effective emission reduction is only determined by restrictions on the rate of temperature change or on the rate of sea-level rise. Limitations on absolute values of temperature and sea-level rise are important only in the long term.

In the next phase of the project, we plan to improve each component of our integrated assessment. An enhanced version of climate response functions and response surfaces will be developed, to clearly distinguish between socioeconomic elements of climate vulnerability and primary impacts of climate change. The climate module will be extended to include other GHGs and sulfur aerosols as well as to provide regional resolution. Finally, work began on a new model of the world economy that seeks to improve several deficiencies of the current models used in integrated assessments.

### End Notes:

<sup>1</sup>According to Tol's concept, this would directly correspond to the corresponding scaling of the physical impacts. Although the functions are derived more or less heuristically, they formally represent a set of static response functions as discussed in the previous subsection. Damages are static as they depend on the state ( $T$ ,  $dT/dt$ ) rather than history. This has been changed later in version 1.5 of the Climate FUND damage module (Tol, 1996).

<sup>2</sup>Note that the damage categories described in Section 3 are algebraic functions of  $T$ ,  $\dot{T} = \mu \ln(1 + C/C_0) - \alpha T$ ,  $S$  and  $\dot{S}$ , and therefore are implicitly included into the general model Eq. (5.1).

<sup>3</sup>In general, there might be more than two extreme values. The algorithm turns out to be rather obvious for our simple model. It gets more complicated and time consuming in more complex models.

<sup>4</sup>Actually, there is no equilibrium due to the long-term melting of the Greenland and West-Antarctic Ice Sheets.

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